Turbulence models

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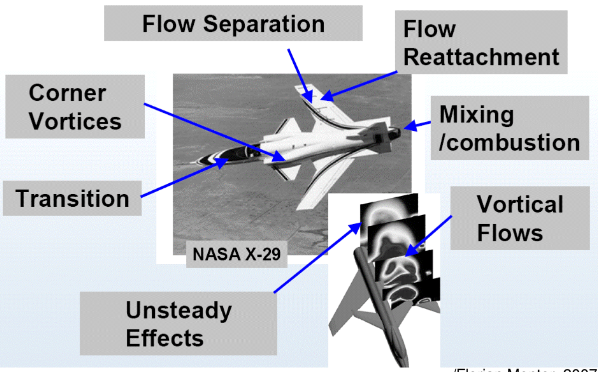
Characteristics of turbulent flows

1. Unsteady, chaotic.
2. Three-dimensional. (Even in 2D flow situations.)
3. Fluctuations are caused by the passing vortices. Advection velocity is the average flow velocity.
4. Turbulence depend not (only) on local flow field but also on the shear-rate history of the fluid parcel.
5. Turbulence causes intensive local mixing of any conserved property. It can be regarded as an increase in transport coefficients.
6. Due to the apparent viscous stresses the kinetic energy of the mean flow is being converted to (stochastic) turbulent kinetic energy and than to internal energy (heating).
7. The size of the largest eddies is close to (and proportional with) the characteristic size of the domain (l).
8. Eddy size cover a wide spectrum.
   $l/\eta=(Re_l)^{3/4}$ - 2..6 orders of magnitude.

Origin of turbulence

1) Wall shear;
2) Free shear;
3) Unstable stratification.

Modeling challenges

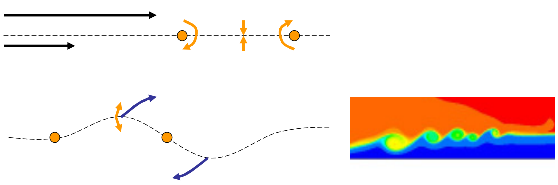


/Florian Menter, 2007/

Free shear layers

Due to the existence of an inflexion point in the velocity profile, a free shear layer is unstable. This can be shown even for 2D flow of perfect fluid. (Kelvin-Helmholtz instability.)

We can model the shear layer with an infinite series of line vortices:



Large eddies create smaller eddies, they create even smaller eddies ...
This turbulent energy cascade is driving kinetic energy from the mean flow to the smallest eddies of η [m] size (dissipative level).

Classification of some well known turbulence models

Algebraic models - Local shear rate + length scale (eg. from wall distance). *Does not know about the flow history, wall distance cannot be defined in complex cases.*

Reynolds averaged (RANS) models based on transport equations:

Spalart-Allmaras	1 eq.	- Airfoils, nearly 2D flow, *Spreading rate of jets are predicted with 100% error.*
k-ε	2 eq.	- For general use 3D, isotropic.
k-ω	2 eq.	- Viscous sub-layer, transition.
RSM	7 eq.	- Anisotropy, eg. for secondary flow and for cyclones. *Up to 10 or 20 times more iterations can be necessary.*

Stabilization of the flow (steady flow) is not guaranteed by any RANS models.

Scale resolving models:

DNS	- Fully resolved turbulence. *Computational cost grows with $Re^{9/4}$. Huge amount of junk data is produced.*
LES,	- Only the large eddies are taken into account. Effect of sub-grid scale turbulence: SGS models. *Close to the wall a fine mesh is required.*
DES, SAS	- RANS model is used close to the wall (e.g.Spalart-Allmaras model), approaches to LES more deeply in the main flow.

Turbulent kinetic energy

The most expressive quantity of turbulence is the turbulent kinetic energy:
$$k=\frac{\overline{u'^2+v'^2+w'^2}}{2}$$ [m²/s²] (Measurable.)

Note that, the square root of k does have the dimension of **m/s**; therefore on the basis of k we can define the velocity scale of turbulence as:

$$V'=\sqrt{k}$$

If isotropic turbulence can be characterized by a single scalar quantity ν_t (turbulent viscosity), which somehow, we need to calculated: ν_t [**m²/s**].

Purely from dimension point of view we need another turbulent quantity having the dimension other than (m/s)ⁿ.

Evolution of *k*

A transport equation for turbulent kinetic energy in general flow situations can be analytically derived. We mention only the two most fundamental source terms below.

$$\frac{dk}{dt}=\underbrace{P}_{\text{production}}-\underbrace{\varepsilon}_{\text{dissipation}}$$

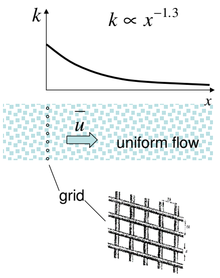
The turbulent production P is interpreted as: $P=\nu_t\,S^2$,

where S is the modulus of the mean rate-of-strain tensor: $S=\sqrt{2S_{ij}S_{ij}}$ $\qquad S_{ij}=\frac{1}{2}\left(\frac{\partial \overline{u_i}}{\partial x_j}+\frac{\partial \overline{u_j}}{\partial x_i}\right)$

Unfortunately we cannot derive a formula for ε.
Some further assumptions have to be made in order to achieve a closure.

Dissipation rate of turbulent kinetic energy

This very fundamental experiment shows the behavior of isotropic turbulence in a „closed system” (without eg. turbulent production in the mean flow).



[From the measurements of Comte-Bellot and Corssin, 1966]

We can define the dissipation rate of turbulent kinetic energy in this experiment as:

$$\varepsilon:=\frac{dk}{dt}$$ [m²s⁻³]

Evolution of ε

According to the **standard *k-ε* model** proposed by Launder and Spalding (1972) ε can be described by a transport equation similar to the equation of *k*, because ε is also rooted in turbulent eddying.

$$\frac{d\varepsilon}{dt}=C_{1\varepsilon}\frac{\varepsilon}{k}P-C_{2\varepsilon}\frac{\varepsilon^2}{k}$$

(We need to correct the dimensions of the production and dissipation terms by multiplying them by ε/k.)

Model constants can be identified on the basis of measured data:

$$C_{1\varepsilon}=1.44,\;\; C_{2\varepsilon}=1.92$$

eg. $C_{2\varepsilon}$ is coming from grid turbulence experiments.

Turbulent viscosity

Assuming that turbulence can be characterized by only 2 scalar parameters k and ε we can define the necessary scales of turbulent motion:

$$T=\frac{k}{\varepsilon}\quad \text{[s]} \quad \longleftarrow \quad \varepsilon=\frac{dk}{dt}$$

$$V'=\sqrt{k}\quad \text{[m/s]} \quad \longleftarrow \quad k=\frac{\overline{u'^2+v'^2+w'^2}}{2}$$

$$L=\frac{k^{3/2}}{\varepsilon}\quad \text{[m]} \quad \longleftarrow \quad L=V'T$$

Now, we can calculate the turbulent viscosity (Kolmogorov-Prandtl formula) :

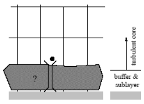
$$\nu_t=C_\mu LV'=C_\mu\frac{k^2}{\varepsilon}$$

From measurements: $C_\mu=0.09$

k-ω model

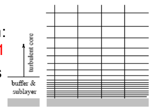
- ω is the second parameter of turbulence (instead of ε)
- ω is proportional to ε/k (eddy frequency)
- Close to the wall ω has better numerical behavior, but in the free stream ε is more suitable.
- The SST model really solves k-ε equations outside of the boundary layer.
- Laminar - turbulent transition can be simulated by k-ω model.
- k-ω model is the platform for further developments in RANS models.

Wall treatment and boundary layer meshing

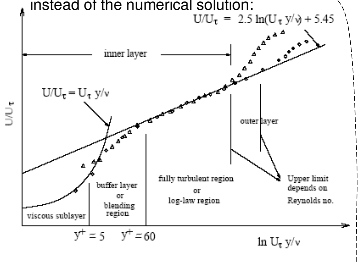


Coarse mesh:
30<y⁺<500
High Re models

Fine mesh:
y⁺=1
Low Re models



Empirical velocity profile (log-law) is used instead of the numerical solution:



Due to the low Re and anisotropic turbulence the turbulent model itself need to be changed.

Model options for low Re wall treetment:

for k-ε model:
Enhanced Wall Treatment

fo k-ω model:
transitional flow

LES

- Some 80% of the total turbulent kinetic energy need to be resolved.
- The minimum grid size for resolving the free stream turbulence is 32^3 . When approaching the wall the eddies are getting smaller, therefore the grid must be refined in every directions.
- Hexahedral mesh is recommended.
- A special (non dissipative) numerical scheme is necessary: Bounded Central Differencing Scheme
- Only 3D, unsteady models are appropriate.
- SGS models, e.g. Smagorinskij-Lilly: $\boxed{\mu_t = \rho L^2 S}$
  (in which L is limited by the cell size and close to the wall it is 0.4y)
- Periodic inlet-outlet or unsteady inlet and non reflective outlet boundary condition is necessary. (There are solutions for unsteady LES inlet BC in FLUENT)

Inlet boundary conditions

Both k and ε (ω) need to be specified.

Turbulent intensity: $I = \frac{u'}{\bar{u}}$

Very silent flow: $I < 1\%$
Very turbulent flow: $I > 10\ \%$
In the core of a channel flow: $I \cong \frac{0.16}{\sqrt[8]{Re}}$

Estimation of the length scale L:
After a perforated plate: hole diameter
Downstream from a small obstacle: height of the obstacle
In the core of a channel flow: 0.07 D

Estimation of some turbulent quantities:

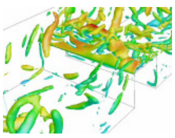
$$\mu_t \cong 1.22\,\rho\,\bar{u}\,I\,L$$

$$k \cong 1.5\,\bar{u}^2 I^2$$

$$\varepsilon \cong C_\mu^{0.75} k^{1.5} L^{-1}$$

$$\omega \cong C_\mu^{-0.25} k^{0.5} L^{-1}$$

Scale resolving models



[LES results from dr. Máté Lohász]

- Unsteady simulations resulting in fluctuating velocity field.
- Less (if any) turbulent viscosity is used, depending on model resolution.
- Relies much less on the accuracy of turbulent models.
- Usually give much better results.
- Synthetic turbulence must to be defined at the inlet.
- Application of special numerical schemes, which do not suppress fluctuations, is necessary.
- Steady field quantities can only be obtained after a long term averaging.