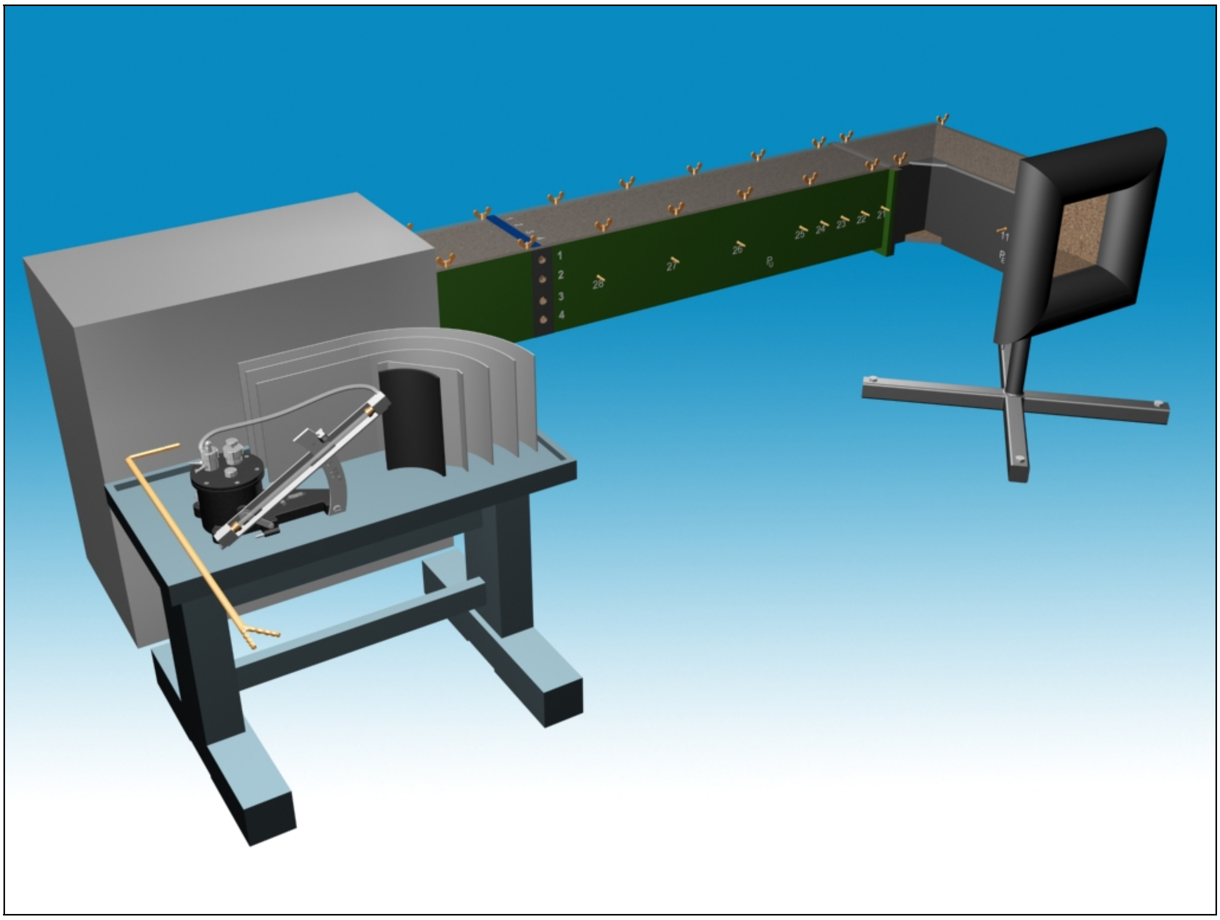


M7

Investigation of the loss coefficient and the flow field of an elbow

1. The aim and practical aspects of the measurement

In pipelines the direction of the flow is usually changed with elbows, i.e. with curved tubes. The change in the direction causes losses, for the reduction of which many methods have been developed. The aim of this measurement is the investigation of the reduction of the losses in an elbow using different geometrical elements. The goal is to find the method providing the best solution.

2. Description of the measurement

The elbow, which is being investigated, has a cross-section of 150×150 mm. A fan which is attached to the end of the duct sucks the air through the system. The air enters the duct upstream of the elbow through a conical inlet orifice. It is assumed that the inlet has minimal losses, and that the air comes from the undisturbed environment. In this way a uniform flow can be realized upstream of the elbow. Downstream of the elbow, static pressure taps can be found along the length of the longer section of the duct. We can examine the flow visually through the plexiglas top of the measurement set-up using pins with long fluffy threads attached to them.

Loss Coefficient

The quality of the elbow is determined by its pressure loss, using the ξ_e loss coefficient. The value of the loss coefficient is influenced by the flow field, with an especially large influence coming from the size of the separation bubbles. In order to guide the air through the bend of the elbow an overpressure is required at the inlet of the duct in order to overcome the secondary losses from the secondary flows and the separation bubbles. If the extent of the secondary losses can be reduced, then the ξ_e can be reduced. This can be achieved by adding different elements into the elbow (Fig. 1) by:

- rounding the inner and outer corners of the elbow with elements having different radii, or by using a 45° plate instead of the sharp corner. Other elements can also be added in the middle of the elbow cross section, which are parallel with the curved walls of the elbow.
- placing different elements (L shaped elements) into the flow, which block the flow in certain areas of the duct and therefore give control over the positioning, size and shape of the separation bubbles. In this way smaller separation zones, having a certain amount of pressure loss associated with them, can be formed, which inhibit the forming of the larger separation zones, and therefore reduce the losses in the duct. In this case the proper size and positioning of the L shaped elements needs to be determined. The different elements can be attached to the wall of the duct both upstream and downstream of the elbow on either side of the channel.

Flow visualization

The measuring apparatus can be used for flow visualization as well. In order to do this either a rod with a long fluffy thread attached to its end can be used, or a number of pins having the same type of threads attached to their ends can be pinned into the cork bottom of the measurement apparatus. In this case, the second possibility will be used. It is advised that a record of the flow fields be kept for the different set-ups, so that these can be included in the laboratory report.

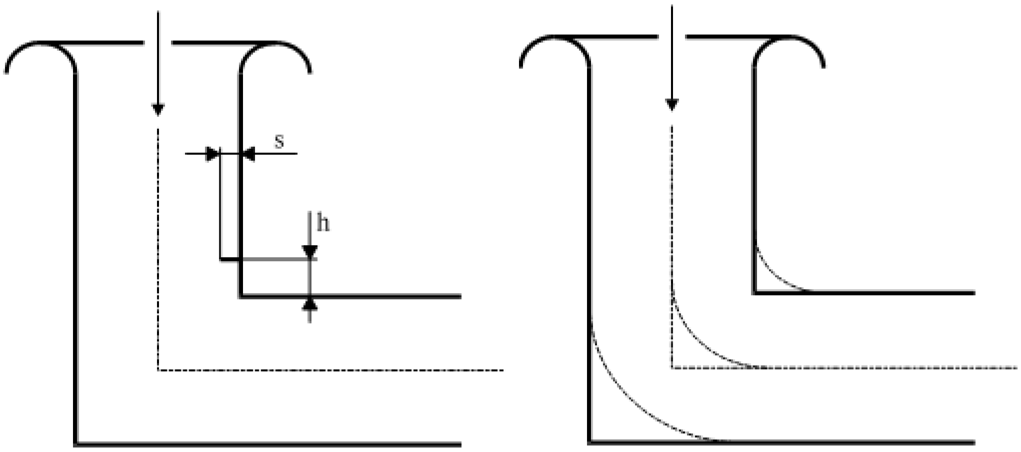
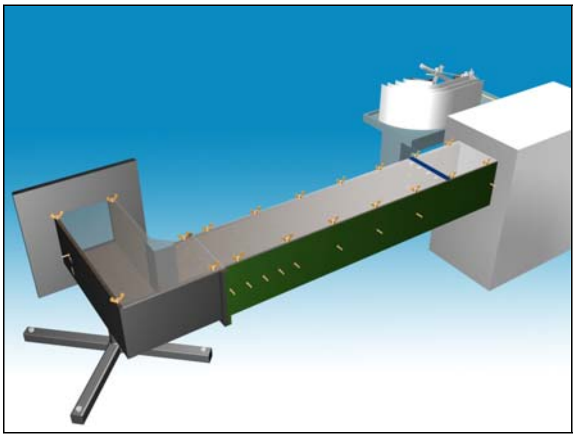
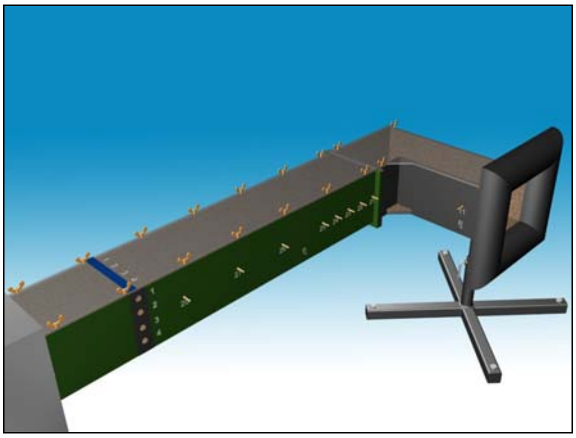


Fig. 1.: ***Different possibilities for manipulating the flow in the elbow.***





3. Evaluation of the measurement results

The loss coefficient can be expressed as the difference between the total pressure upstream ($p_{t,b}$) of the bend and the total pressure downstream ($p_{t,a}$) of it divided by the dynamic pressure (p_{dyn}).

$$\xi_e = \frac{p_{t,b} - p_{t,a}}{\frac{\rho}{2}\bar{\mathrm{v}}^2}$$

where ρ is the density of the fluid, and $\bar{\mathrm{v}}$ is the average velocity in the cross-section of the duct.

The average velocity can be determined by using a Prandtl tude, otherwise known as a Pitot-static tube, and the point to point measurement technique. In this case the measurements should be made upstream of the bend in 2x2=4 points. Note though, that the $\bar{\mathrm{v}}$ can change with the introduction of the different elements into the flow.

During the measurement, the first step is to determine the volume flow rate using the dynamic pressure measured using the Prandtl tube.

$$q_v = A \cdot \bar{v} = L^2 \cdot \frac{\sum_{i=1}^{4} v_i}{4} = L^2 \cdot \frac{\sum_{i=1}^{4} \sqrt{\frac{2 \cdot p_{dyn,i}}{\rho}}}{4}$$

This will be used as a calibration value, by which we will determine the value of the *k* correction factor. Once we have calculated the volume flow rate using the Prandtl tube, we need to use the digital manometer in order to measure the pressure difference between the pressure tap which can be found right after the inlet to the duct and the ambient pressure, Δp_{meas}. This pressure difference gives a good approximation of the dynamic pressure experienced in the duct. A correction factor must be applied to this value though, in order to account for the wall friction and the inlet losses.

The volume flow rate calculated from the difference in static pressure:

$$q_v = k \cdot A \cdot \sqrt{\frac{2}{\rho} \cdot \Delta p_{meas}}$$

where k is the correction factor: $k = \dfrac{L^2 \cdot \dfrac{\sum_{i=1}^{4}\sqrt{\dfrac{2\cdot p_{dyn,i}}{\rho}}}{4}}{A\cdot\sqrt{\dfrac{2\cdot \Delta p_{meas}}{\rho}}} = \dfrac{\sum_{i=1}^{4}\sqrt{\dfrac{2\cdot p_{dyn,i}}{\rho}}}{4\cdot\sqrt{\dfrac{2\cdot \Delta p_{meas}}{\rho}}}$

Knowing the correction factor, the use of the Prandtl tube is no longer necessary. The volume flow rate can now be calculated using the Δp_{meas} and the k.

In order to calculate the loss coefficient, we need to calculate the total pressure difference: ($p_{t.b}$-$p_{t,a}$). On the other hand, since the cross-section of the duct is the same before as well as after the bend, the dynamic pressure is going to be the same in both cross-sections because of continuity. Therefore the equation for the loss coefficient can be reduced in the following matter, with the static pressure difference appearing in the numerator and the dynamic pressure in the denominator.

$$\xi_k = \frac{p_{t,b} - p_{t,a}}{\frac{\rho}{2}\overline{v}^2} = \frac{(p_{st,b} + p_{dyn,b}) - (p_{st,a} + p_{dyn,a})}{\frac{\rho}{2}\overline{v}^2} = \frac{(p_{st,b} + \frac{\rho}{2}\overline{v}^2) - (p_{st,a} + \frac{\rho}{2}\overline{v}^2)}{\frac{\rho}{2}\overline{v}^2} = \frac{p_{st,b} - p_{st,a}}{\frac{\rho}{2}\overline{v}^2}$$

During the measurement, we can directly measure the pressure difference by connecting the one end of the manometer to the tap which is upstream of the bend and the other one to the taps which are downstream of the bend.

$p_{st,b}$: The upstream static pressure tap is obvious in the measurement set-up. Here the upstream effect of the elbow onto the flow can be neglected.

$p_{st,a}$: In choosing the downstream measurement point, the main question is, how far does the disturbing effect of the elbow influence the measurement, and therefore which measurement point should be used? After the bend in the duct, the two sidewalls have 8-10 taps placed at a certain interval, which can be used for measuring the static pressure. In examining the streamlines and the pressure reading results, it can be found, where the flow separation on the wall of the duct reattaches (using the flags), which should coincide with the cross-section where the pressure readings on both sides of the duct are approximately the same, due to the almost parallel streamlines. It should be noted though, that since the velocity profiles at this cross-section are not yet entirely symmetric, we are looking for the cross-section where the measured static pressure does not increase further. Here it can be said that the flow has reattached to the wall of the duct and that the velocity profile is once again symmetric. This is the static pressure which should be considered as the downstream value of the static pressure, and with which the loss coefficient of the elbow should be calculated.

/The two sets of pressure taps which are closest to the fan do not give accurate readings due to the inlet to the fan, and therefore should not be considered during the measurement./

The ξ_e needs to be examined for different set-ups, with the goal being the attainment of a $\xi_{e,\min}$ value.

4.The report should contain the following:

- Pitot-static tube measurement data, and the value for the average velocity and calibration constant.
- A table containing the static pressure values, the pressure distribution along the wall of the duct, as a function of the length (x[mm]).(The positions of the different pressure taps should be measured!) A diagram with the two separate pressure distributions on the two sides of the duct, which should be made for every set-up.
- Drawings of the different flow visualization results for the different set-ups.
- It needs to be determined where the flow reattaches to the wall of the duct, and therefore, where the velocity profile is symmetric. The loss coefficient for the particular set-up needs to be calculated at this point.
- Results should be presented for all the examined cases in a similar manner.
- The different set-ups should be compared, with the results being given in a tabular or graphical manner, pointing out the most advantageous and the disadvantageous set-ups. Do not forget to examine the basic empty duct with the 90° elbow, which is the reference duct for the measurements. By comparing the loss coefficients of the different set-ups to these, we can arrive at different conclusions for our results.
- There should be an individual evaluation made of all the different results.
- Error calculations should be made for the measurement results of the loss coefficient, based on what is assigned.

Error calculation

Loss coefficient of the elbow:

$$\zeta_k = \frac{\Delta p_t}{\frac{\rho}{2}\bar{\mathrm{v}}^2} = \frac{p_{st.b} - p_{st,a}}{\frac{\rho}{2}\left(\frac{\mathrm{q_v}}{\mathrm{A}}\right)^2} = \frac{p_{st,b} - p_{st.a}}{\frac{\rho}{2}\left(\frac{\sum_{\mathrm{i}=1}^{16}\mathrm{A_i}\cdot \mathrm{v}_i}{\mathrm{A}}\right)^2} = \frac{p_{st.b} - p_{st.a}}{\frac{\rho}{2}\left(\frac{\sum_{\mathrm{i}=1}^{16}\mathrm{v}_i}{16}\right)^2} = \frac{p_{st.b} - p_{st.a}}{\frac{\rho}{2}\left(\sqrt{\frac{2p_{dyn}^*}{\rho_l}}\right)^2} = \frac{p_{st.b} - p_{st.a}}{p_{dyn}^*}$$

where p_{dyn}^* is the dynamic pressure, which is calculated back from the $\bar{\mathrm{v}}$ average velocity.

The absolute error calculation: $\delta\zeta_e = \sqrt{\sum_{i=1}^{n}\left(\delta X_i \cdot \frac{\partial \zeta_e}{\partial X_i}\right)^2} = ?$

where X_i is the measured amount and the measurement error associated with it:

$X_1 = p_{t,b} - p_{t,a} = p_{st.b} - p_{st.a} = \Delta p_{\mathrm{st}}$ Error of the digital manometer: $\delta p_{instrument} = 2Pa$

$X_2 = p_{dyn}$ Error of the digital manometer: $\delta p_{instrument} = 2Pa$

The relative error calculation: $\frac{\delta\zeta_e}{\zeta_e} = ?$

In the laboratory report, both the relative and absolute errors need to be given for the loss coefficient.

Remember that during the labs:

- Before turning any measurement device on or in general during the lab, make sure that safe working conditions are ensured. The other participants have to be warned of the starting of the machines and of any changes that could endanger the members of the lab.
- The atmospheric pressure and room temperature should be recorded before and after every measurement.
- The measurement units and other important factors (e.g. data sampling frequency, date of calibration) of every recorded value of the applied measurement devices should be recorded.
- Type and construction number of the applied measuring instrument should be included in the final report.
- Checking and harmonizing of the units of the recorded values with those used in further calculations.
- Manometers should be calibrated if necessary.
- The measurement ports of the pressure meter should be carefully connected to the correct pressure ports of the instrument.
- If inlet or outlet tubes are to be assembled with fans, connections should be airtight as escaping/entering air can significantly modify the measurement results.

Bibliography:

Lajos Tamás: Az áramlástan alapjai, Budapest 2008, (10. fejezet Hidraulika)