<u>Technical Acoustics and Noise Control</u> *(lecture notes for self-learning)*

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Lecture 9.

Content:

9.1. Energetics of acoustic waves, single characteristic value for energetic relations

To characterise the sound field, the time functions of the acoustic variables contain detailed information. So many data at certain simple calculations are not required, it is enough to know any single characteristic value, for example the average of them. This chapter will summarise the calculation method of these values.

Average sound intensity: The time averaged sound intensity can calculate as, the time weighted mean value,

$$\bar{I} = \frac{1}{T}\int_0^T I' dt$$

In the integral expression the T is average time, for a periodic case it is the time of period, for a non-periodic occurrence T should be a value, that does not effects the result of the integral.

The average intensity of harmonic sound waves: In optional case, a phase shift should appear between the sound pressure and particle velocity. Let be at the x=0m distance the sound pressure and particle velocity time functions,

$$p'(t) = \hat{p}\, cos(\omega t - \varphi_1) \quad \text{, and} \quad v'(t) = \hat{v}\, cos(\omega t - \varphi_2)$$

Let apply the definition, the time average of the sound intensity is,

$$\bar{I} = \frac{1}{T}\int_0^T I' dt = \frac{1}{T}\int_0^T p'v' dt = \frac{1}{T}\int_0^T \hat{p}\, cos(\omega t - \varphi_1)\ \hat{v}\, cos(\omega t - \varphi_2) dt =$$

$$= \frac{1}{T}\int_0^T \frac{\hat{p}\hat{v}}{2}\left(cos(2\omega t - \varphi_1 - \varphi_2) + cos(\varphi_2 - \varphi_1)\right) dt =$$

The first term inside the bracket is periodic, so the average is zero, the rest is,

$$\bar{I} = \frac{\hat{p}\hat{v}}{2} cos(\varphi_2 - \varphi_1)$$

The average value depends on the amplitudes and the phase difference of the components parallel. In a free field plane wave sound propagation the phase difference is zero. But for not plane waves or in the vicinity of objects, blocking the sound propagation, the phase difference may be between 0 and π/2 radian, that highly effects the average value.

The average intensity in complex exponential representation: When calculating the average, the direct product of the sound pressure and particle velocity exponential terms leads to bad result. The fault originated from the imaginary "stowaway" component. The fault can correct, to take the complex conjugated of one member of the product,

$$\bar{I} = \frac{1}{2} Re(\boldsymbol{p}'\boldsymbol{v}'^{*}) = \frac{1}{2} Re\left(\hat{p}e^{-i\varphi_1}e^{i\omega t}\;\hat{v}e^{i\varphi_2}e^{-i\omega t}\right) = \frac{1}{2} Re\left(\hat{p}\hat{v}e^{i(\varphi_2-\varphi_1)}\right) = \frac{\hat{p}\hat{v}}{2} cos(\varphi_2 - \varphi_1)$$

Average intensity for free field plane wave sound propagation: The sound pressure and the particle velocity are in phase at free field plane wave sound propagation. Let express the particle velocity from the algebraic form of linear acoustic equation of motion, and put it in the expression of the intensity,

$$I' = p'v' = p'\frac{p'}{\rho_0 a} = \frac{p'^2}{\rho_0 a}$$

The average intensity for a free field plane wave sound propagation,

$$\bar{I} = \frac{1}{T}\int_0^T I'dt = \frac{1}{T}\int_0^T p'v'dt = \frac{1}{T}\int_0^T \frac{p'^2}{\rho_0 a}dt = \frac{p_{eff}^2}{\rho_0 a}$$

Where the effective or RMS (root mean square) sound pressure value is,

$$p_{eff} = p_{RMS} = \sqrt{\frac{1}{T}\int_0^T p'^2 dt}$$

Effective value for harmonic time dependence,

$$p_{eff} = \sqrt{\frac{1}{T}\int_0^T p'^2 dt} = \sqrt{\frac{1}{T}\int_0^T (\hat{p}\; cos\; \omega t)^2 dt} = \sqrt{\frac{\hat{p}^2}{T}\int_0^T \frac{1+\cos 2\omega t}{2}dt} = \sqrt{\frac{\hat{p}^2}{2T}\left[t + \frac{\sin 2\omega t}{2\omega}\right]_0^T} =$$

After the ω=2π/T replace, the relation between the effective value and amplitude is,

$$p_{eff} = \frac{\hat{p}}{\sqrt{2}}$$

The instantaneous sound power: To characterise a sound source, the instantaneous sound power can calculate by the surface integral of the instantaneous sound intensity,

$$P' = \int_A \underline{I}'d\underline{A}$$

Between the vector terms, dot product is needed, because only the intensity vector component parallel with the surface element vector will pass through the surface. The instantaneous sound power as a function of the area (A) and the perpendicular (normal) surface average instantaneous sound intensity (I'_{na}),

$$P' = I'_{na} A$$

The average sound power: To characterise a sound source, the time averaged sound power can calculate by the surface integral of the average sound intensity,

$$P = \int_A \underline{\bar{I}}\, d\underline{A}$$

The time averaged sound power as a function of the area (A) and the perpendicular (normal) surface and time averaged sound intensity ($\bar{I}_{na}$),

$$P = \bar{I}_{na} A$$

9.2. Levels in acoustics

In physics all variables, that are power, proportional with power, or in a power term proportional with power can express in level. The level expression of the ξ "power-like" physical variable,

$$L_\xi = 10\, lg \frac{\xi}{\xi_0} \quad [dB]$$

Where ξ_0 is the reference value, making the argument of 10 base logarithm dimensionless. If ξ_0 is the international standardised value, the L_ξ is an absolute level. When ξ_0 is optional, the L_ξ is a relative level. The measurement unit of the level is the one tenth bel, the decibel, with abbreviated form dB (honour to Alexander Graham Bell, Scottish borne, American scientist, inventor and engineer). The advantages of the level are to compress the numeric scale (think about the limited size scale of an old fashioned measurement devices), and the logarithm change much more close to the subjective sound sensation characteristic than the linear one. The level is not a real physical variable, so in every case, when we use it, the real physical variable and its reference value must be noted.

In airborne acoustics generally used levels are the sound power level (to characterise the sound source), the sound intensity level (to characterise the sound propagation) and the sound pressure level (to characterise the sound field). The definitions are,

Sound power level: $L_W = 10\, lg \frac{P}{P_0}$ [dB] , where $P_0 = 10^{-12}$ [W]

Sound intensity level: $L_I = 10\, lg \frac{I}{I_0}$ [dB] , where $I_0 = 10^{-12}$ [W/m^2]

Sound pressure level: $L = 10\, lg \frac{p_{eff}^2}{p_0^2}$ [dB] , where $p_0 = 2 \cdot 10^{-5}$ [Pa]

The level representation characteristically change the numeric value of the acoustic variables. To improve the numeric sense, sound power and sound power level values of sound sources and effective sound pressure and sound pressure level values of sound fields are listed in the following tables.

Name of the sound source	P [W]	L_W [dB]
Silent whisper	10^{-10}	20
Everyday conversation	10^{-7}	50
Loud shout	10^{-3}	90
Pianoforte (fortissimo)	0,1	110
Acoustic organ (romantic)	10	130
Civil defence siren	100	140
Space rocket at launch	10^{7}	190

Sound powers and sound power levels of typical sound sources

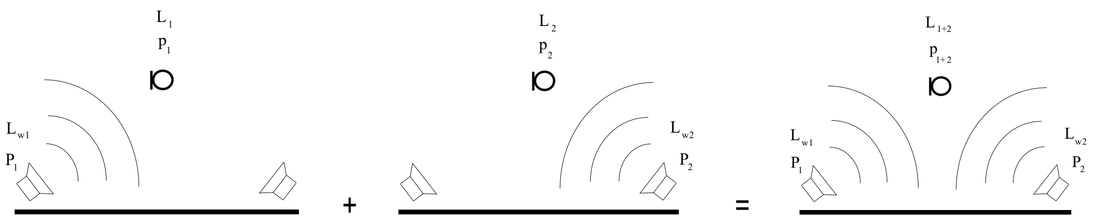
Name of the sound field	p_{eff} [Pa]	L [dB]
Threshold of hearing (1 kHz-en)	$2\ 10^{-5}$	0
Background noise in a sound studio	$2\ 10^{-4}$	20
Silent living area at night	$2\ 10^{-3}$	40
Normal human speech 5m far	$2\ 10^{-2}$	60
Busy traffic road on pavement	0,2	80
Petrol chain saw in operator position	6,3	110
Airplane jet engine 10m far	63	130

Effective sound pressures and sound pressure levels of typical sound fields

Mathematic operations with levels:

General rule that the mathematic operation with levels will governed by the mathematic operations with the original physical variables and the logarithmic rules together. Often used operations are summation, subtraction, multiplication and division.

The sum of levels:



The notation applied to calculate the resultant sound pressure level of two sound fields

The sound pressure time function, the sound pressure effective value and the sound pressure level at the x= 0m position observation point, created by the 1. loudspeaker,

$$p'_1(t) = \hat{p}_1\, cos(\omega_1 t)\ ,\quad p_{eff1} = \frac{\hat{p}_1}{\sqrt{2}}\ ,\quad L_1 = 10\, lg\frac{p^2_{eff\,1}}{p_0^2}$$

The sound pressure time function, the sound pressure effective value and the sound pressure level at the x= 0m position observation point, created by the 2. loudspeaker,

$$p'_2(t) = \hat{p}_2\, cos(\omega_2 t + \Delta\varphi)\ ,\quad p_{eff2} = \frac{\hat{p}_2}{\sqrt{2}}\ ,\quad L_2 = 10\, lg\frac{p^2_{eff\,2}}{p_0^2}$$

The amplitudes, the angular frequencies and the initial phases of the components are different. To determine the resultant sound pressure level (L_{1+2}), we have to know the resultant effective sound pressure, to determine the resultant effective sound pressure (p_{eff1+2}) we have to know the resultant sound pressure (p'_{1+2}) of the component sound fields at x=0m position. Referred to the principle of the linear superposition, the resultant sound pressure is the simple algebraic sum of the component sound pressures.

$$p^2_{eff\ 1+2} = \overline{(p'_{1+2})^2} = \overline{(p'_1 + p'_1)^2} = \overline{\left(\hat{p}_1\, cos(\omega_1 t) + \hat{p}_2\, cos(\omega_2 t + \Delta\varphi)\right)^2} =$$

$$= \overline{\left(\hat{p}_1^2\, cos^2(\omega_1 t) + 2\hat{p}_2\hat{p}_2\, cos(\omega_1 t)cos(\omega_2 t + \Delta\varphi) + \hat{p}_2^2\, cos^2(\omega_2 t + \Delta\varphi)\right)} =$$

To ease the calculation of averages the square and cross product terms should convert to linear expressions,

$$= \overline{\left(\hat{p}_1^2\,\frac{1+cos2\omega_1 t}{2} + 2\hat{p}_1\hat{p}_2\frac{1}{2}\big(cos(\omega_1 t+\omega_2 t+\Delta\varphi)+cos(\omega_1 t-\omega_2 t-\Delta\varphi)\big) + \right.}$$

$$\overline{\left. + \hat{p}_2^2\,\frac{1+cos2(\omega_2 t+\Delta\varphi)}{2}\right)} =$$

We supposed that the angular frequencies of the component sound waves are different ($\omega_1 \neq \omega_2$), so the average all of the periodic (cosine) term will be zero. The rest will be,

$$p_{eff\ 1+2}^2 = \frac{\hat{p}_1^2}{2} + \frac{\hat{p}_2^2}{2} = p_{eff\ 1}^2 + p_{eff\ 2}^2$$

The resultant sound pressure level,

$$L_{1+2} = 10\ lg\frac{p_{eff\ 1+2}^2}{p_0^2} = 10\ lg\left(\frac{p_{eff\ 1}^2}{p_0^2} + \frac{p_{eff\ 2}^2}{p_0^2}\right) = 10\ lg(10^{0,1L_1} + 10^{0,1L_2})$$

Comments:

- When summarising sound fields, for different frequency sounds, the effective sound pressure square of the resultant sound field is equal with the simple algebraic sum of the effective sound pressure square of the component sound fields. The principle of linear superposition is true for acoustic energy too.

- When summarising more than two sound fields, completing the sum with par-to par, the summed series can increase with optional elements,

$$p_{eff\ 1+2+\cdots n}^2 = p_{eff\ 1}^2 + p_{eff\ 2}^2 + \ \dots + p_{eff\ n}^2$$

$$L_{1+2+\cdots n} = 10\ lg(10^{0,1L_1} + 10^{0,1L_2} + \ldots + 10^{0,1L_n})$$

- When summarising two identical sound pressure level ($L_1 = L_2$), the resultant level will increase with 3 dB to one of the original,

$$L_{1+2} = 10\ lg(10^{0,1L} + 10^{0,1L}) = 10\ lg(2\cdot 10^{0,1L}) = 10\ lg(2) + 10\ lg(10^{0,1L}) = 3 + L$$

- When the difference of the added sound pressure level is 10 dB, the resultant sound pressure level will increase with 0,4 dB. Concerning the normal requirement of acoustic calculations and measurements accuracy, this value is usually negligible,

when $L_2 - L_1 \geq 10\ dB$, $L_{1+2} \approx L_2$

- For average accuracy practical technical acoustic problems, the measurement and calculation results usually will be rounded to one decimal place.

- When the angular frequencies of the summarised sounds are equal ($\omega_1 = \omega_2$), than the effective sound pressure square of the resultant sound field will be effected by the effective sound pressure square of the components and the phase difference as well ($\Delta\varphi$),

$$p_{eff\ 1+2}^2 = p_{eff\ 1}^2 + p_{eff\ 2}^2 + 2p_{eff\ 1}p_{eff\ 2}cos\Delta\varphi$$

The multiplication and division of levels (amplification and attenuation):

Ducted silencers often are applied to reduce the transmitted noise in a channel. The silencer will reflect or absorb a part of the incident sound energy, and only the rest can pass. The attenuation coefficient (ζ_s), the ratio of the transmitted and incident acoustic energy, is an important value to characterise the effectiveness of the silencer,

$$\zeta_s = \frac{p_{eff\ tr}^2}{p_{eff\ in}^2}$$

Let express the transmitted component,

$$p_{eff\ tr}^2 = \zeta_s\ p_{eff\ in}^2$$

With level representation,

$$10\ lg\frac{p_{eff\ tr}^2}{p_0^2} = 10\ lg\left(\zeta_s \frac{p_{eff\ in}^2}{p_0^2}\right) = 10\ lg\zeta_s + 10\ lg\left(\frac{p_{eff\ in}^2}{p_0^2}\right)$$

$$L_{tr} = L_{in} + \Delta L_{\zeta s}$$

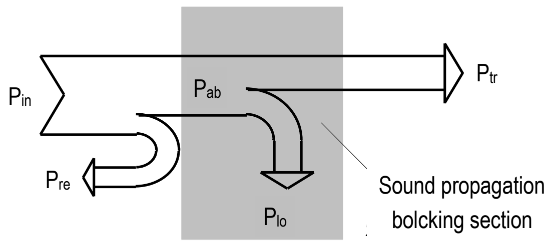
Where the sound pressure level measured inside the duct before the silencer (L_{in}), the sound pressure level measured inside the duct after the silencer (L_{tr}), and the difference of them is the noise reduction of the silencer (ΔL_{ζ}). Instead of a silencer let take an optional acoustic system unit, the general phenomena of ζ is,

$$L_{tr} = L_{in} + \Delta L_{\zeta} \quad \text{, where} \quad \Delta L_{\zeta} = 10\ lg\zeta$$

The meaning of ζ is, attenuation when $0 \le \zeta < 1$, and amplification when $1 < \zeta$.

9.3. Single values to characterise the sound propagation

An acoustically non transparent object, that blocks the sound propagation, will disperse and, or absorb the incident sound wave. The dispersion (or scattering) includes the phenomena of reverberation, refraction and diffraction. An object, that blocks the sound propagation could be, a wall separates rooms, a car exhaust silencer, dense forest lane, or a highly turbulent layer of air flow, etc. The sound incident to the blocking object will decomposes to back reflected, absorbed, loss and transmitted components. The notation of the sound power delivered by the different components, can find the on the band diagram.



Components of the sound passing through a blocking section

The dimensionless ratios to characterise the energetic relations of the sound propagation,

Absorption coefficient: $\alpha = \frac{P_{ab}}{P_{in}}$

Reflection coefficient: $r = \frac{P_{re}}{P_{in}}$

Loss coefficient: $\delta = \frac{P_{lo}}{P_{in}}$

Transmission coefficient: $\tau = \frac{P_{tr}}{P_{in}}$

The transmission coefficient has elevated importance in engineering noise control, so taking into consideration practical aspects, we express it in level as well, that is called transmission loss (R),

$$R = 10\ lg\frac{1}{\tau} \quad [dB]$$

The power ratios to characterise the energetics of the sound propagation are physically simple phenomena. But relating them, there are some practical disadvantages. For example the difficulties during experimentally determining their values. The power cannot measure directly. Another problem is, that in a lot of practical cases the sound passes from the source to the observer on a multiple way. One power ratios can characterise only one of these, but we need the resultant value. To solve this problems, in the practical engineering noise control we introduced the insertion loss and noise reduction single value variables. These are physically composed phenomena, but easy to measure them and summarising the effects of the different propagation paths.

Insertion loss: The insertion loss (ΔL_{il}) is the difference of the sound pressure levels measured in the same point without and with the sound propagation blocking object ($L_{without\ blocking}$ and $L_{with\ blocking}$),

$$\Delta L_{il} = L_{without\ blocking} - L_{with\ blocking}$$

Arrangement to determine the insertion loss of an object

Noise reduction: The noise reduction (ΔL_{nr}) is the difference of the sound pressure levels measured before and after the sound propagation blocking object (L_{before} and L_{after}),

$$\Delta L_{nr} = L_{before} - L_{after}$$

Arrangement to determine the noise reduction of an object

To determine the insertion loss or noise reduction it is enough the measure two sound pressure level, which is usually not complicated. Both of these variables include the resultant effects of the different sound propagation paths. For example if the blocking object is a ducted silencer, the sound can pass the silencer directly through air (poor air borne sound propagation), but in the metal wall of the silencer too (air borne - structure borne – air borne sound propagation). One concrete insertion loss or noise reduction value will be true only for one concrete acoustic arrangement.

9.4. Test questions and solved problems

T.Q.1. Write down the phenomena of the instantaneous and averaged sound intensity, and the calculation method for a ω angular frequency harmonic sound wave!

T.Q.2. List the levels applied in acoustics, and show the rules of mathematic operations with levels!

T.Q.3. Prove that the resultant effective sound pressure square of the different frequency sound components is the simple sum of the components effective sound pressure square!

S.P.1. Noise emission of a water chiller in operation is planned to determine with onsite measurement. The measurement is disturbed by a background noise, that is impossible to stop. In the measurement point the resultant sound pressure level of the water chiller and background noise is 75dB. After switch off the chiller, the sound pressure level of the poor background noise is 71dB. Calculate the sound pressure level of the water chiller operational noise, independently from the background noise in the test point!

$$p_{eff\ b+c}^2 = p_{effb}^2 + p_{eff\ c}^2$$

$$p_{eff\ c}^2 = p_{eff\ b+c}^2 - p_{effb}^2$$

$$L_c = 10\ lg(10^{0,1L_{b+c}} - 10^{0,1L_b}) = 10\ lg(10^{7,5} - 10^{7,1}) \approx 72,8\ dB$$
