Questions from
"Advanced Fluid Mechanics"

Dr. Gergely Kristóf

23 May 2009

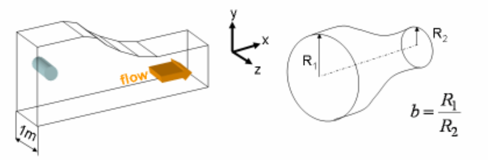
---

1. Vortical flows

Derive the 2D form of the vorticity transport equation from the Navier-Stokes equation!

Derive the evolution equation of the elementary fluid line, and show it's analogy with the vorticity transport. What practical conclusions can be drawn?

Problem #1.1



Compare a 2D confuser (of slab symmetry) with an axial symmetric confuser:

- What components of the vorticity vector are non-zero? Use cylindrical coordinates (x,r,φ) in the axisymmetric case!
- In what proportion does the length of a fluid element change?
- In what proportion will the components of the vorticity change if the vortex diffusion is negligable?

For what reasons can the vorticity of a fluid parcel change in incompressible flow? Please classify the terms.

2. Irrotational flows

How can the pressure field be calculated from the velocity field, in the case of irrotational flows of an ideal fluid and in the case of Darcy flow?

From what equation can the velocity potential of a constant density irrotational fluid flow be obtained? What is the velocity potential for a point source of Q [m^3/s] intensity?

Define the vector potential for the velocity field of a constant density fluid flow! Show its relation to the stream function and explain the physical meaning of the stream function.

Derive the Poisson equation for the stream function! Define the complex potential and prove that the Couchy-Riemann conditions are fulfilled!

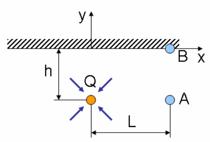
Compute the velocity field and the circulation Γ for the free vortex characterized by $w=iC_0 ln(z)$!

Problem #2.1

What is the shape of the water surface above the drain of the bath tub? Determine the drop of water level between points characterized by r_1 and r_2!

$v_z \approx 0$ the field variables depend only on r.

Problem #2.2



a. Construct the complex potential for this flow! (Q, h and L are given.)
b. Determine the velocity magnitude in B!
c. What is the volume flow-rate between A and B?
d. Calculate the pressure distribution along axis x for Darcy flow of a given permeability and viscosity!

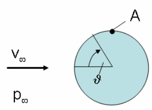
Problem #2.3

What is the shape of the streamlines close to a stagnation line? y=f(x)

Problem #2.4

a) Prove that the streamlines are circular, and touching upon the x axis from the positive y direction, in the origin of the coordinate system!
b) Please, sketch the streamlines!

Problem #2.5



a. Calculate v_A for a given v_∞!
b. Determine the distribution of the pressure coefficient over the surface of the cylinder: v=f(ϑ).

Problem #2.6

What circulation intensity is necessary for shifting the stagnation point by ϑ_0 angle?

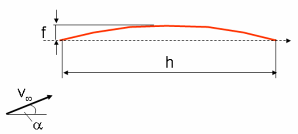
Problem #2.7

Please, specify the equation of a circle around the complex point ε,

passing through the real point $\sqrt{a_{10}}$.

Problem #2.8

Estimate the lift coefficient (c_l) of an arched plate!
α and f/h can be regarded as given values, with both being small.



$$c_L = \frac{F_L}{\frac{\rho}{2} v_\infty^2 A}$$

3. Boundary layers

Define the displacement thickness! How is it related to the boundary layer thickness for laminar flow past a flat plate of zero angle of inclination? Derive the relation between δ/x and Re_x for laminar boundary layers!

Problem #2.1

Compare the critical value of Re_δ (corresponding to laminar-turbulent transition) for a flat plate and in a circular pipe by assuming:

$$\delta \cong \frac{D}{2}$$

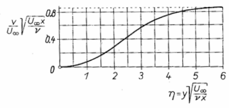
Problem #2.2

Please, estimate the order of magnitude of each term in the dimensionless continuity, and in the dimensionless equation of motion of a steady boundary layer flow!

Show the self-similar form of the boundary layer equation for laminar flow!

Problem #2.3

Please, calculate the displacement velocity v(x,δ)
(y velocity profile at the edge of the boundary layer)
over a flat plate of zero inclination for given l, Re_l and U_∞.



What effects can help the transition from laminar to turbulent boundary layer?

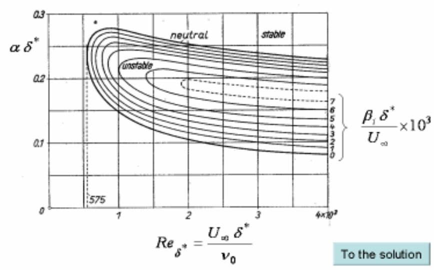
Problem #2.4

Please, calculate the vorticity of the perturbation velocity field for Tollmien-Schlichting waves!

Sketch the stability map of the boundary layer flow past a flat plate with 0 pressure gradient. Define the dimensionless quantities appearing on both coordinate axis.

Problem #2.5

Please, calculate the displacement thickness and the wavelength of highest amplification factor for a flat plate of zero inclination at Re_x=20000, x=0.1 m.
(This is roughly a speed of 108 km/h in standard atmosphere.)



To the solution

Express the turbulent viscosity ν_t on the basis of the assumptions of Prandtl's mixing length theory!

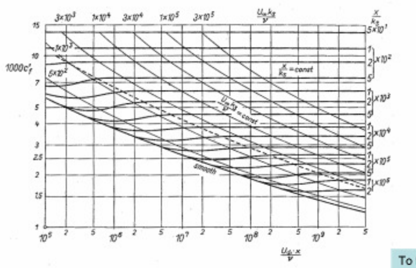
Derive the velocity profile for the laminar sub-layer and for the fully turbulent layer! Describe the structure of the turbulent boundary layer!

Problem #2.6

Determine the turbulent viscosity ratio (ν_t / ν_0) in the logarithmic layer for a given value of y^+!

Problem #2.7

Determine the maximum magnitude of sand roughness for which a flat plate can be regarded as hydraulically smooth. The free stream velocity and the kinematical viscosity are given: $U_\infty = 15 \text{m/s}, \quad \nu_0 = 1.5\times10^{-5}\,\text{m}^2\text{s}^{-1}$



To the solution

Explain the numerical method for solving the boundary layer equation of turbulent flow?

What requirements must airfoils fulfill? What methods can be used for delaying the transition and for avoiding BL separation?

5. Turbulence models

What are the major characteristics of turbulent flows?

Define the turbulent kinetic energy dissipation for grid turbulence! How can the time, velocity, and length scales be defined on the basis of k and ε? Specify the formula for the turbulent viscosity!

Specify the evolution equations of k and ε (mention only the most fundamental terms)!

Describe the main characteristics of algebraic, RANS, and scale resolved models!

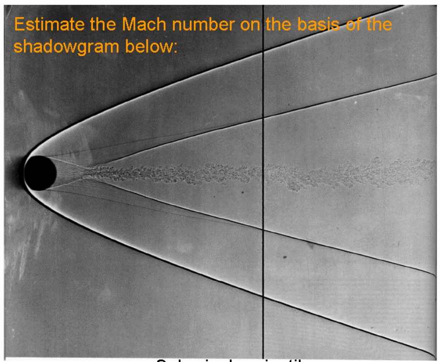
6. Gas dynamics

Derive the formula for calculating the speed of sound in ideal gases!

Explain the formation of shock waves from a series of small compression waves! What are the major characteristics of shocks?

Problem #6.1

Estimate the Mach number on the basis of the shadowgram below:



[An album of fluid motion] Spherical projectile

To the solution

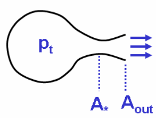
Derive the relation between the relative velocity increase (*dv/v*) and the relative increase of the channel cross-section (*dA/A*)!

Derive the relation between the temperature ratio (T_t/T) as a function of the Mach number (M) for an isentropic flow!

Problem #6.2

Please, calculate the maximum velocity for isentropic flow if γ=1.4, R=287 J/kg-K and T_t=1000 K are given!

Problem #6.3



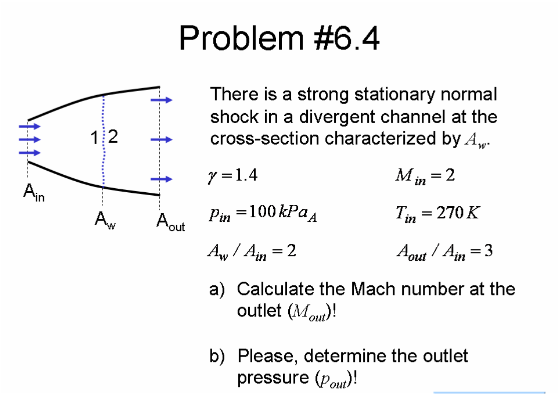
a) What is the optimum A_{out}/A_* ratio of the nozzle of a rocket thruster designed for near ground flight, if the chamber pressure p_t=10 bar_A , and γ=1.3. Please, use the gas tables!

b) Calculate the mass flow-rate for T_t=1300 K a, R=462 J/kg-K and A_{out}=20 cm^2!

c) Please, calculate the thrust!

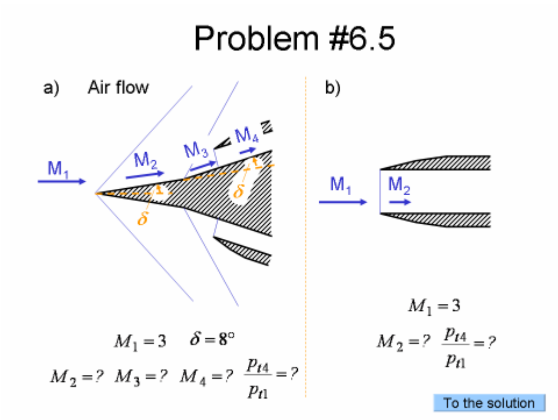
Derive the quadratic equation for the square of upstream and the downstream Mach numbers from the conservation laws applied to a steady normal shock!

Draw qualitatively correct graphs of the pressure, density, temperature, Mach number and stagnation pressure ratios for a normal shockwave!

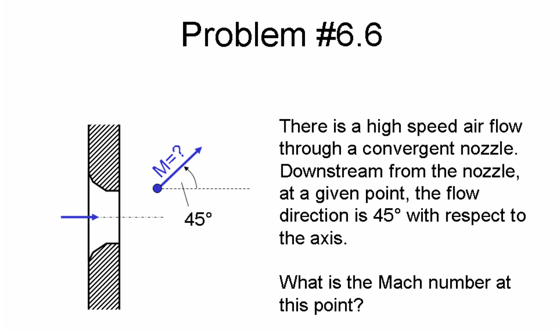


What are the major differences between a Mach wave and an oblique shock? Prove that the tangential velocity component does not change and that the normal velocity component will change according to the laws valid for normal shocks!

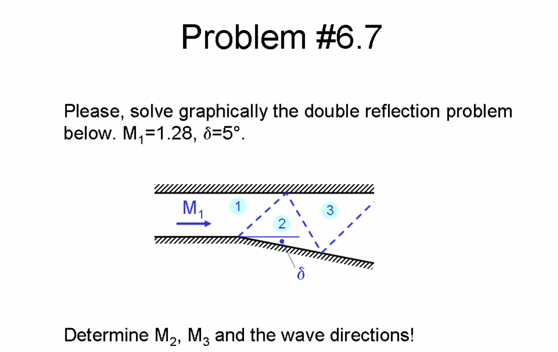
Draw the qualitatively correct contour graph of the change of the angle of the flow direction (δ) as a function of the upstream Mach number (M_1) and the angle of the oblique shock (β)! What conclusions can be drawn from this graph?



Derive the integral formula for the δ angle as a function of the Mach number for the Prandtl-Meyer expansion!



Derive the relation between M and M* and the integral formula for the δ angle as a function of M*. What type of curve is it and how can it be used for the graphical solution of expansion waves and weak shocks on the hodograph plane?



Explain what kinds of waves are formed in supersonic flow past corners and curved surfaces: How can the reflection of a shockwave be avoided by the proper surface shaping? Is it possible to create an isentropic compression by a curved surface? What are the limitations of this?

Explain the dynamics of pressure and velocity fluctuations in under-expanded and over-expanded supersonic jets!

Show the variation of pressure in a convergent-divergent nozzle as a function of the streamwise coordinate for different pressure ratios!

What is a shock tube? How will the pressure, temperature, and velocity change shortly after the start of the experiment?

7. Atmospheric flows

Problem #7.1

Please, calculate the pressure profile for a given (linear) temperature profile:

$$\bar{T} = T_0 - \gamma z \; ,$$

in which $\gamma = -\dfrac{\partial T}{\partial z} = \text{const.}$.

Draw the graphs of the standard (ICAO) atmospheric temperature profile and the potential temperature profile. Explain why the standard atmosphere is stable!

Derive the vertical component of the equation of motion for the perturbation quantities in atmospheric flow using the Boussineq approximation for the buoyancy force!

Specify the x,y,z components of the Coriolis force in atmospheric flow! Which terms are negligible and why? Show the relation between the pressure gradient and the wind direction for a geostrophic wind!

Problem #7.2

a) Calculate the below non-dimensional pressure gradient for a gradient wind in cylindrical system of coordinates:

$$\frac{\dfrac{\partial p}{\partial r}}{\rho f v_g}$$

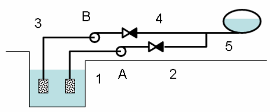
r is the distance from the center of the cyclone and $v_g = f(r)$.

b) What is the magnitude of the non-dimensional pressure gradient for a geostrophic wind?

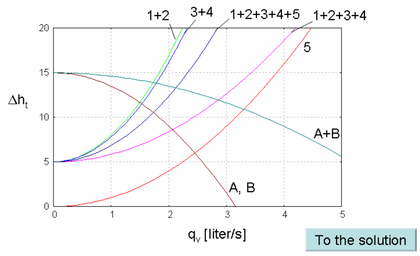
Draw the Ekman spiral for a medium latitude case on the Northern hemisphere!

8. Hydraulics

Problem #8.1



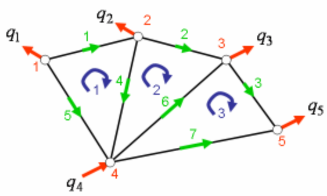
a) Explain how the characteristic curves could be combined!
b) What is the head loss of conduit #5?
c) What total heads are produced by pump A and pump B?



What are the advantages of looped distribution networks? What laws can be employed for the hydraulic analyses of a looped network?

Problem #8.2

a) Specify the loop matrix for the pipe network bellow:



b) Construct the loop equation for loop 1using constant indices (1,4,5) for the unknown volume flow-rates.

Describe the Cross method and derive the loop correction formula!

Derive the reduced elasticity modulus for liquids contained by thin walled pipelines!

Problem #8.3

Compare the wave celerity in still water with those in a pipeline of given geometrical parameters:

Pipe diameter: 500 mm,
Wall thickness: 10 mm,
E_{water}: 2.0 x 10^9 Pa,
E_{steel}: 2.1 x 10^{11} Pa.

Derive the characteristic variables from the 1D form of the continuity equation and the equation of motion (including wall friction) by applying acoustic assumptions.

Describe the method of characteristics! What are the boundary conditions for a closed end, for an open end and for a pipe junction?

Problem #8.4

We suddenly open one end of an evacuated pipe.
What will be the pressure and inflow velocity immediately after the opening?
Please, use the method of characteristics and calculate α, β quantities!
Define the initial state of the pipe on the basis of v=0, p=const. conditions.

Pressure in the closed pipe:	50 kPa,
External pressure:	100 kPa,
Air density:	1.2 kg/m^3 ,
Sound speed:	334 m/s.