

Technical Acoustics and Noise Control (lecture notes for self-learning)

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Lecture 11.

Content:

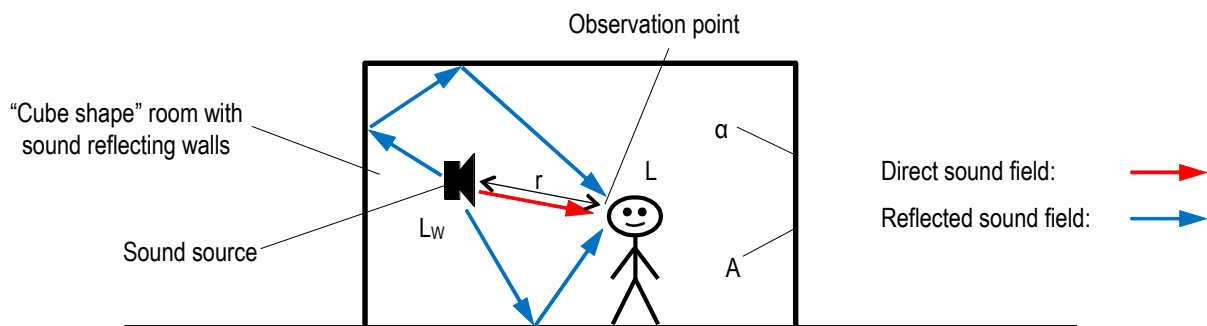
11.1. The model of a sound field bounded with sound reflecting surfaces (lecture notes)

11.2. Test questions and solved problems

11.1. The model of a sound field bounded with sound reflecting surfaces

Sound sources several times operate in closed space, so parallel with the free space, another important problem in engineering noise control is to determine the noise exposure of a sound source placed in an enclosure bounded with sound reflecting surfaces. The wave acoustic modelling applied in acoustics (to derive the wave equation and its solution for the concrete problem) can apply in rooms as well. But this is quite complicated even for a simple case. It is needed a model, that is fast, easy to use, can handle the complicated room interior and accurate enough.

The theoretic background of the model is the ray acoustic modelling. In ray acoustics the wave nature of the sound is limited, the sound is taken as energy transporter train, that can converge or diverge, and when arrives a border surface, partly absorbed and reflected. In a homogeneous medium, that are in static rest the sound rays are straight lines. So get away from the source the cross sectional area increase of the ray channel can calculate easily (see: sound field of a point source in free field). This model can apply in closed space too, with completion, when the sound ray channel reaches the wall, one part of the incident acoustic energy will absorb (and leave the internal sound field), the other part will reflect back (remain in the internal sound field). So in an optional observation point in the room direct and reverberated sound ray components will appear.



Direct and reverberated sound fields in a room bounded with sound reflecting surfaces

If the boarder surface is a good sound reflective, a big portion of the sound power will exist inside the room even after many reflection. The multiple reverberations will build up the reflected sound field. The reflected sound field increases the volumetric energy density of the direct sound file in the room. In case of a cube shape room, good sound reflecting surfaces, omnidirectional sound source radiating continuously in time, the direction of the rays and sound energy density will be well balanced, and so called, diffuse sound field will be formed. The simplifications during the derivation are the same, applied in linear acoustics (the medium is a homogeneous, inviscid and non heat conducting, continuum, isentropic elementary thermodynamic process, the medium is in static rest and the amplitudes are small), the source is an omnidirectional point source, and calculation is valid far from the origin of the sound. Let say something about the new simplifications (cube shape room, good sound reflecting surface and diffuse sound field).

- For an acceptable diffusivity among the internal sizes of the room the biggest size should not exceed the length of three times the smallest. For a , b and c room internal envelope sizes,

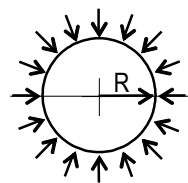
$$a \geq b \geq c \quad , \quad \text{a room is acoustically "cube shaped" if} \quad 1 \leq a/c \leq 3$$

- To form a diffuse sound field it is required good sound reflecting surfaces inside the room. If the walls are sound absorber, a big part of the acoustic energy will be absorbed, and the reverberated sound field will be weak.

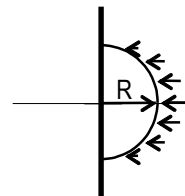
- In a perfectly diffuse sound field the acoustic volumetric energy density is independent from the position and the time average of the instantaneous sound intensity vector will be zero (for every intensity vector will appear an opposite pair in a short time).

$$e_v(\underline{r}) = \text{const.} \quad , \quad \text{and} \quad \bar{\underline{I}} = \frac{1}{T} \int_0^T \underline{I}'_v dt = 0$$

Excluding the close vicinity of the sound source in a cube shape room with good sound reflecting surface the continuous operation of an omnidirectional point source will form a diffuse sound field. For the later calculation it is important to know the ratio between the average reverberated sound intensity inside a perfectly diffuse sound field and the average intensity perpendicular to the wall.



The incident intensity vector distribution on the reference ball inside the diffuse sound field



Near the wall the perpendicular projection of the intensity vectors incident on the half reference ball in a diffuse sound field

The sound power on the surface of (R) radius ball incident from the diffuse sound field

Take an attention to the drawing above, the ratio of the sound powers incident to the total ball surface inside the diffuse sound field and to the half ball surface placed on the wall by the perpendicular components are the same than the ratio of areas of the total ball surface and perpendicular projection surface of the ball (the same radius circle). Let introduce a fictive radiation area (A), the ratio of the average reverberated sound intensity inside a perfectly diffuse sound field (I_r) and the average intensity perpendicular to the wall (I_{rpw}),

$$\frac{P_{tb}/A}{P_{hbp}/A} = \frac{I_r}{I_{rpw}} = 4 \quad I_{rpw} = \frac{I_r}{4}$$

The sound field of a point source placed in a room bounded with sound reflecting surfaces:

The goal of the mathematic model is to determine the sound pressure level (L), created by a (L_w) sound power level point source placed in a room bounded with sound reflecting surfaces. Inside the room the sound source will create a direct and reverberated sound field. The resultant total intensity (I_t) in an optional observation point the simple algebraic sum of the direct sound field (I_d) and the reverberated sound field (I_r)

$$I_t = I_d + I_r$$

For the direct sound field, the relation among the sound power (P) radiated by the point source, the distance between the source and observer (r), directivity coefficient (D) and the intensity of the direct sound field (I_d) is,

$$I_d = \frac{PD}{4\pi r^2}$$

where,

$$D = \frac{A_{sphere\ total}}{A_{sphere\ real}}$$

Switch on the sound source and after the transient building up of the reverberated sound field, the sound power, enters the reverberant sound field (P_{rin}) is equal with the sound power, that leaves the reverberant sound field (P_{rout}),

$$P_{rin} = P_{rout}$$

The sound power, that enters the reverberant sound field (P_{rin}) given by the first reflection from the wall, the product of the sound power (P) and averaged reflection coefficient (\bar{r}). The directly radiated sound power (P) was taken into consideration at the direct sound field, so the sound power, enters the reverberant sound field must be the power after the first reverberation. The sound power, that leaves the sound field (P_{rout}) is the product of the perpendicular component of the reverberated sound intensity (I_{rpw}), the internal surface of the room (A) and the surface weighted average sound absorption coefficient ($\bar{\alpha}$),

$$P\bar{r} = P(1 - \bar{\alpha}) = I_{rpw}A\bar{\alpha}$$

where,

$$\bar{r} = 1 - \bar{\alpha}$$

Let replace the perpendicular component of the reverberated sound intensity (I_{rpw}) with the average reverberated sound intensity (I_r), and express it,

$$I_r = \frac{P(1 - \bar{\alpha})}{4A\bar{\alpha}} = \frac{4P}{\frac{A\bar{\alpha}}{1 - \bar{\alpha}}} = \frac{4P}{R_c}$$

Where the room constant (R_c) is,

$$R_c = \frac{A\bar{\alpha}}{1 - \bar{\alpha}}$$

and the surface weighted average sound absorption coefficient,

$$\bar{\alpha} = \frac{\sum_{i=1}^n \alpha_i A_i}{\sum_{i=1}^n A_i}$$

Where (n) is the number of the surfaces with different sound absorption coefficient in the room, (α_i) is the sound absorption coefficient of the i -th surface section, and (A_i) is the area of the i -th surface section. Usually the sound absorption coefficient is the function of the frequency (f). The resultant total intensity (I_t) formed by the sound power (P) of a point source in a room with sound reflecting surfaces is,

$$I_t = I_d + I_r = \frac{PD}{4\pi r^2} + \frac{4P}{R_c} = P \left(\frac{D}{4\pi r^2} + \frac{4}{R_c} \right)$$

Far enough from the source, where the sound propagation can approximate as plane wave propagation, and the sound pressure and the particle velocity are in phase, the total intensity (I_t) can change to the formula with effective sound pressure (p_{eff}),

$$\frac{p_{eff}^2}{\rho_0 a} = P \left(\frac{D}{4\pi r^2} + \frac{4}{R_c} \right)$$

Let's apply level notation,

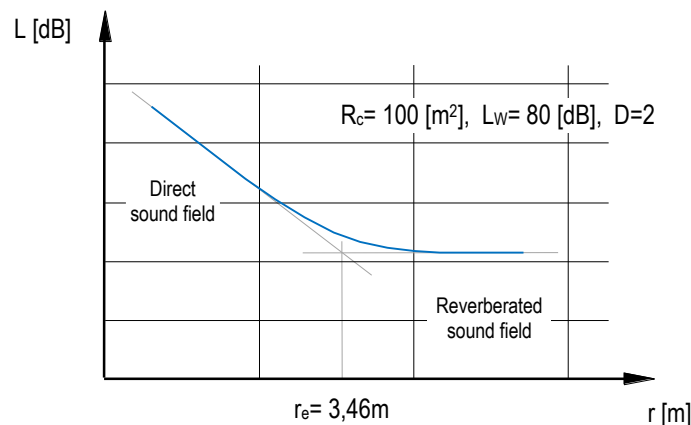
$$10\lg \frac{p_{eff}^2}{(2 \cdot 10^{-5})^2} + 10\lg \frac{400}{\rho_0 a} = 10\lg \frac{P}{10^{-12}} + 10\lg \left(\frac{D}{4\pi r^2} + \frac{4}{R_c} \right)$$

The first term at left is the sound pressure level (L), the second in technical normal state air ($t_0=20^\circ\text{C}$, $p_0=1\text{bar}$, $\rho_0 a \approx 408 \text{ [kg/m}^2\text{s]}$) approximately negligible. The second term at right is the sound power level (L_w). The (L) sound pressure level (r) meter far from the sound source in a reverberant sound field is,

$$L = L_w + 10\lg \left(\frac{D}{4\pi r^2} + \frac{4}{R_c} \right) \text{ [dB]}$$

Comments:

- The sound pressure level (L) that can measure in a sound field bounded with sound reflecting surfaces depend on the emitted sound power level (L_w), the distance between the sound source and the observer (r), the directivity coefficient (D), and the room constant (R_c).
- Because of the presence of the reverberated sound field, at same radiated sound power (P) and distance (r) a bigger sound pressure level (L) will form in a space bounded with sound reflecting surfaces, than in free space.
- In the audible frequency region the loss of the reverberated sound field mainly reasoned by the absorption on the walls and not the dissipation in air.
- Close to the sound source the direct sound field is dominant. For a point source double the distance from the origin of the radiation a 6dB sound pressure level decrease will take place.
- Far from the sound source the reverberated sound field is dominant. In the reverberated sound field the sound pressure level (L) is independent from the distance (r).



The sound pressure level (L) as a function of the distance between the point source and observer in a room bounded with sound reflecting surfaces (at constant L_w , D and R_c) and the radius of the equivalent energy sphere surface radius (r_e)

- On the room acoustic equivalent energy surface (that is a (r_e) radius sphere surface originated in the place of the point source) the sound intensity of the direct sound field and the reverberated sound field are equal to each other (this is important for speech intelligibility).

11.2. Test questions and solved problems

T.Q.1. Define the phenomena of perfectly diffuse sound field. Let derive the sound pressure level (L) of a point sound source with (L_w) radiated sound power level, placed in a reverberant room! Detail the simplifications!

S.P.1. Determine the A-weighted sound pressure level in the observation point positioned 8m far from a point source, placed on the ground level in a 20m x 13m base area and 8m high workshop hall! The sound power radiated by the point source (P_{oct}), the sound absorption coefficient (α_{oct}) and the relative levels of the A-weighting (ΔL_{Aoct}) as a function of the octave-band frequencies (f_{oct}) can be found in the following tabulation.

f_{oct} [Hz]	250	500	1k	2k	4k
P_{oct} [W]	0.1	1	0.1	0.01	0.001
α_{oct} [-]	0.05	0.03	0.02	0.02	0.02
ΔL_{Aoct} [dB]	-8.6	-3.2	0	1.2	1

Solution:

f_{oct} [Hz]	250	500	1k	2k	4k
$L_{\text{Woct}} = 10 \lg(P_{\text{oct}}/P_0)$ [dB]	110	120	110	100	90
$R_{\text{Toct}} = A\alpha_{\text{oct}}/(1-\alpha_{\text{oct}}) = 1048 \cdot \alpha_{\text{oct}}/(1-\alpha_{\text{oct}})$ [m^2]	55.2	32.4	21.4	21.4	21.4
$L_{\text{oct}} = L_{\text{Woct}} + 10 \lg(D/4\pi r^2 + 4/R_{\text{coct}}) = L_{\text{Woct}} + 10 \lg(2/4\pi 5^2 + 4/R_{\text{coct}})$ [dB]	99.0	111.1	102.9	92.9	82.9
$L_{\text{oct}} + \Delta L_{\text{Aoct}}$ [dB]	90.4	107.9	102.9	94.1	83.9

$$A = 2 \cdot ((20+13) \cdot 8 + 20 \cdot 13) = 1048 \text{m}^2$$

$$L_A = 10 \lg(10^{9.04} + 10^{10.79} + 10^{10.29} + 10^{9.41} + 10^{8.39}) \approx 109.3 \text{ [dB(A)]}$$

(Note: See details about the A-weighting in the measurement syllabus and more details later in the lecture notes)

S.P.2. In a pure reverberant sound field, made by a sound source placed in a room, the sound pressure level is 84 dB. One third part of the internal surface of the room is changed to a sound absorbing material. After this the sound pressure level decreased to 78 dB. Calculate the sound absorption coefficient of the absorbing material, if the sound absorption coefficient of the room internal surface, α_{room} is 0.05.

Solution:

$$\Delta L = 6 = L_1 - L_2 = L_{W1} - L_{W2} + 10 \lg(4/R_{T1}) - 10 \lg(4/R_{T2}) = 10 \lg(R_{T2}/R_{T1}) =$$

$$10 \lg(\alpha_2 A(1 - \alpha_1)/\alpha_1 A(1 - \alpha_2)) = 10 \lg(\alpha_2(1 - 0.05)/0.05(1 - \alpha_2))$$

$$\text{So, } \alpha_2 \approx 0.1732$$

$$\text{and } \alpha_2 = (\alpha_1/3 + \alpha_{\text{absorber}}/3)/A = 0.05 \cdot 2/3 + \alpha_{\text{absorber}}/3 \approx 0.1732$$

The sound absorption coefficient of the absorbing material, $\alpha_{\text{absorber}} \approx 0.42$
