Technical Acoustics and Noise Control *(lecture notes for self-learning)*

Dr. Gábor KOSCSÓ   titular associate professor (BME Department of Fluid Mechanics)

Lecture 2.

Content:

2.1. Irregular mathematic introduction: In an engineering approach, mathematics is a method by which we can gain knowledge about phenomena without having to perform the phenomenon itself in physical reality. Furthermore, with numbers we can give the exact extent of things. Instead of small and large markers, we can prescribe the size of the variables just needed for the intended processes. To understand this, let is take a simple example well known in mechanical engineering practice. Our task is to make a gas-filled, 2 m^3, cylindrical, 1 m diameter tank with an internal gauge-pressure of 20 bar. In a conventional approach, the most important property required to ensure stability is the tank wall thickness. If the wall thickness were to be determined by trial, without preliminary calculations, the task could only be solved by dangerous, expensive and time-consuming work. In contrast, the minimum wall thickness formula derived by mathematical modelling of the problem prepares the concrete practical implementation under safe, cheap, and fast conditions. Without the derivation from statics, well known to all mechanical engineers, in this case using only the end result,

$$\delta = \frac{\Delta p D}{2\sigma_h} = \frac{20 \cdot 10^5 \cdot 1}{2 \cdot 200 \cdot 10^6} = 0{,}005\ [m]$$

At a maximum allowable mechanical stress of 200 MPa, without safety factor and corrosion reserve, the minimum tank wall thickness required to ensure stability is 5 mm. To determine the required wall thickness to solve the static problem, it took less time to understand and apply the mathematical model, but presumably even to guess it, than to perform a series of stability tests with containers with different wall thicknesses under real conditions.

It is important to note that in the mathematical modelling of a phenomenon, the analysis of the solution may reveal new details about the phenomenon that are not detected by the superficial experimental study. Thus, it can be stated, that mathematical modelling serves not only engineering design but also scientific understand. Countless examples can be listed in physics (particle physics, astronomy, …), when a new recognition was born based on a mathematical model and the experimental validation of the theoretical discovery took place much later. A detailed examination of the solutions of the mathematical model is also useful during learning. It systematizes our knowledge, draws attention to hidden details and, together with the initial conditions, also shows the limits of application. Thus, the examination of solutions is also used in the presentation of acoustic phenomena in the chapters of this lecture notes.

Steps of mathematic modelling: Selection of physical variables describing the phenomenon, selection of physical principles related to the phenomenon, writing of mathematical equations describing the physical principles and making connections between the variables, simplification, solution and control. In the next section, we will go through these steps in detail to model sound phenomena. The starting point is helped by the flow nature of airborne sounds, i.e. the sound is a specific compressible fluid flow, so the variables and basic equations used in fluid mechanics can be applied to create the mathematical model air borne sound acoustics.

Variables describing the sound space:

- Velocity vector, $\underline{v}$ [m/s] (distance vector per unit time), (the quantity of the vector is indicated by underlining the letter symbol of the variable)

- Pressure, p [Pa] (normal force per unit area)

- Density, ρ [kg/m^3] (mass per unit volume of a material)

- Temperature, T [K] (degree of warming of materials, quantity proportional to the average kinetic energy resulting from the random heat motion of the material particles)

Physical principles related to sound phenomena:

- Material or mass conservation (no material or mass is formed or lost during flow or sound occurrence)

- Momentum balance (the change of the momentum of a fluid particle per unit time is equal to the resultant forces, acting on it)

- Energy balance (the change of the energy of a medium per unit time is equal to the sum of the work (power) per unit time done by the resultant forces and the supplied heat per unit time)

- Laws on physical properties of materials (principle determining the behaviour of the medium in relation to the phenomenon, law of matter, e.g. the ideal gas law, Hooke's law, or in another fields of physics, e.g. Ohm's law)

Basic equations to describe sound phenomena:

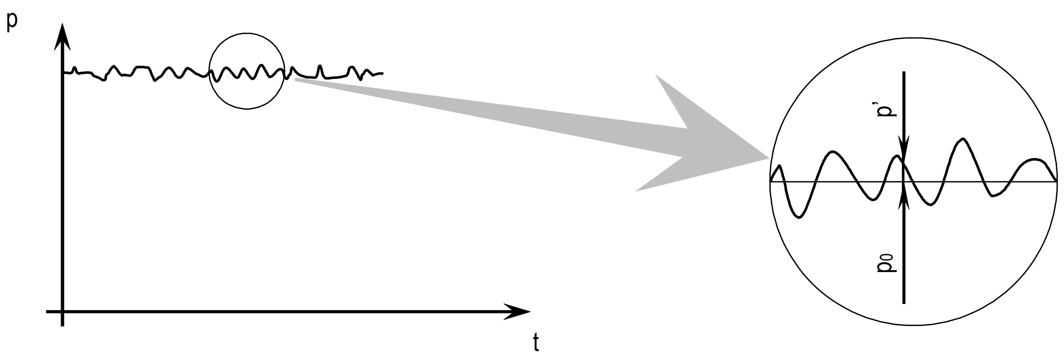
- Continuity equation (to express the principle of material or mass conservation)

- Equation of motion (to express the momentum balance)

- Energy equation (to express energy balance)

- Equation of state for ideal gases (in most cases the noise transmitted to the listener by air, on the other hand the sound are formed in a gas, can be considered ideal from a thermodynamic point of view, so in our case the law of material is the equation of state for ideal gas)

Resolution of acoustic variables into equilibrium and time-varying parts: It is an empirical observation that variables describing sound field can be decomposed into time-constant, large equilibrium values and time-varying, small values (see the following figure). We can also say that acoustics is the science of very small changes in very large quantities. We will experience several advantages of the resolution in later derivations (e.g.: the temporal derivatives of the equilibrium terms will be eliminated, the quadratic expressions of the time-varying terms or their products will be negligible).



The resolution of pressure, describing sound field, into equilibrium and time-varying parts

$$p = p_0 + p' \qquad \underline{v} = \underline{v}_0 + \underline{v}' \qquad T = T_0 + T' \qquad \rho = \rho_0 + \rho'$$

For sound field variables, the left side of the expressions denotes the total quantity without the index, the index "0" denotes the time-constant, equilibrium term, and the dash (comma) denotes the time-varying, fluctuating part (not the local derivative!). Due to their prominent importance in acoustics, the variables p' and $\underline{v}'$ were given separate names, as sound pressure (p') and particle velocity ($\underline{v}'$).

Simplification conditions:

- The medium giving space for sound propagation is a homogeneous, frictionless, continuum (homogeneous: material properties are independent of location, frictionless: parallel liquid layers can be moved relative to each other without resistance, continuum: the material structure of the medium is continuous and non-atomic or molecular).

- The elemental thermodynamic state changes (processes) in the medium during sound propagation will take place without heat exchange (adiabatic) and without loss (dissipation), i.e. in an isentropic way (p/ρ^{κ}= const.).

- Sound propagation occurs in a medium in static rest ($\underline{v}_0$= 0 m/s)

- Except particle velocity, for all of the acoustic variables it is true that the ratio of time fluctuation to equilibrium value is much less than one,

$$p'/p_0 \ll 1 \qquad T'/T_0 \ll 1 \qquad \rho'/\rho_0 \ll 1$$

2.2. Direct algebraic relationship of variables describing sound field: To establish our acoustic calculations, the first step is to examine the relationship between acoustic variables, without space and time dependence, using algebraic equations. To do this, suppose a very simple case of sound propagation (see the following figure). The middle of a very long tube, filled with air is blocked by a perfectly sealed, frictionless movable piston. At time t_0, the piston starts to move at a speed v' from left to right. As a result, mechanical waves start in the air from both sides of the piston. Now let limit our studies to the wave component moving to the right. Thanks to the simple task selection, a one-dimensional plane wave propagation parallel to the tube axis is created. When solving the problem, by choosing a coordinate system moving together with the wave front, the phenomenon becomes constant in time, and the space dependence is also eliminated, because there are only two (undisturbed and disturbed) sound field variables at the two side of the wave front. Our conservation and balance equations are written on a control surface, with elementary thickness, including the wave front entirety, moving at the speed of sound "a" and permeable to the medium (see the following figure).

Continuity equation: In the case of a time-steady channel flow, the mass flow entering the front face of the control surface is equal to the mass flow exiting the back face,

$$q_{m\,in} = q_{m\,out}$$

The mass flow is the magnitude of the mass flowing over a given surface (A) per unit time, generally the surface integral of the product of the density (ρ) and velocity ($\underline{v}$). The operation between velocity and the surface element vector ($d\underline{A}$) is scalar (dot) product,

$$q_m = \int_A \rho \underline{v}\, d\underline{A}$$

In the inlet and outlet cross sections, the surface element vector and the velocity vector are parallel to each other at each point, and in each cross section the magnitude of the density and velocities are constant, so that the mass flows can be determined by simple products,

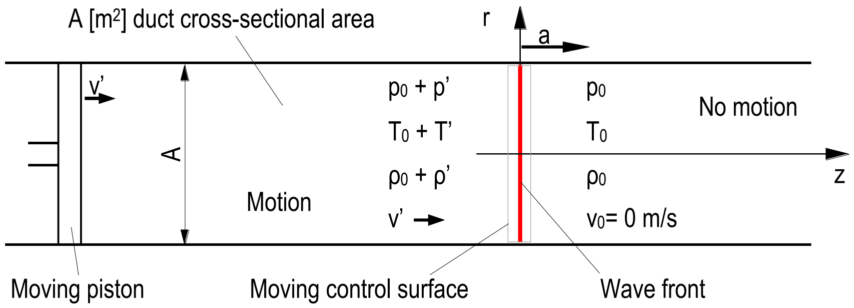
$$\rho_0 A a = (\rho_0 + \rho')A(a - v')$$

Simplified by the cross-section and after opening the bracket,

$$\rho_0 a = \rho_0 a + \rho' a - \rho_0 v' - \rho' v'$$

After the simplification, neglecting the small term in the second order (ρ'v'), the algebraic shape of the acoustic continuity equation expressed on the particle velocity,

$$v' = \frac{a}{\rho_0} \rho'$$



Examination of a one-dimensional disturbance wave in a coordinate system moving with the wave front

Equation of motion: The change in the momentum of a fluid particle per unit time is equal to the resultant forces acting on the fluid particle (Newton's second law). In the case of a steady flow in time, the change in momentum is the difference between the momentum flowrate entering and leaving the front and back of the control surface. The change in the momentum flowrate is caused by the result of the pressure forces acting on the control surface,

$$\dot{I}_{in} - \dot{I}_{out} = F_{p\ out} - F_{p\ in}$$

If the left side is positive, the momentum flowrate within the control surface decreases, the magnitude of which is determined by the difference between the pressure forces acting on the inlet and outlet side. The momentum flowrate vector is the momentum passing over a given surface per unit time, the product of the mass flow and the velocity (together with the fluid, not only the mass, but also the momentum of the fluid passing through the control surface), in the general case,

$$\underline{\dot{I}} = \int_A \underline{v} \rho \underline{v}\, d\underline{A}$$

Where the product $\rho \underline{v} d\underline{A}$ is the elementary mass flowrate. Furthermore, the pressure force, is the surface integral of the pressure (p),

$$\underline{F}_p = -\int_A p\, d\underline{A}$$

With constant density and pressure and when the velocity of the flow is parallel to the surface element vectors, the integral expressions of the momentum flowrate and the pressure forces are simplified into products,

$$\rho_0 A a^2 - \rho_0 A a (a - v') = (p_0 + p')A - p_0 A$$

On the left side of the equation, in the second term, the term inside the bracket is the mass flowrate out of the control surface (q_{mout}), which is replaced by the formally simpler inlet mass flowrate (q_{min}), based on the continuity equation. After performing the simplifications, the algebraic form of the acoustic equation of motion expressed in terms of sound pressure,

$$p' = \rho_0 a v'$$

Energy equation: The change in the energy of a fluid part per unit time is equal to the work done by the forces acting on the fluid part per unit time. Due to the adiabatic change of state and the frictionless condition, the heat supplied to the fluid is zero. In a time steady flow, the energy change is the difference between the energy flowrates exciting the back and entering the front of the control surface. The change in energy flowrates is the result of the work performed per unit time (power) by the pressure forces acting on the control surface,

$$\dot{E}_{out} - \dot{E}_{in} = \dot{W}_{Fp\ in} - \dot{W}_{Fp\ out}$$

If the left side is positive, the energy flow within the control surface increases, the magnitude of which is determined by the difference of the work done per unit time by the pressure forces on the upstream and downstream side. The energy (E) of the fluid part is the sum of the kinetic, potential and internal energies (E_k, E_p and E_i). In a homogeneous air space, the external forces acting on the fluid particles and the pressure forces are balanced. In other words, the air particles float in their own medium, so to move them, do not have to work against the external force field, and there is no change in potential energy. Energy flowrate is the magnitude of the energy passing through a given surface per unit time, the product of the mass flowrate and the energy per unit mass (e). Not only the mass of the fluid, but also the energy of it passes through the surface. In the general case, the sum of the kinetic and internal energy flowrate,

$$\dot{E} = \int_A e\, \rho \underline{v}\, d\underline{A} = \int_A (e_k + e_i) \rho \underline{v}\, d\underline{A} = \int_A \left(\frac{v^2}{2} + c_v T \right) \rho \underline{v}\, d\underline{A}$$

Where the $\rho \underline{v} d\underline{A}$ product is the elementary mass flowrate, e_k is the kinetic energy and e_i is the internal energy per unit mass, c_v is the specific heat at constant volume and T is the absolute temperature of the medium. Power is the work done per unit time,

$$\dot{W} = \frac{dW}{dt} = \frac{d(\underline{F} d\underline{s})}{dt} = \underline{F} \frac{d\underline{s}}{dt} = \underline{F}\, \underline{v}$$

Work done by the pressure force per unit time on the fluid particles passing through an arbitrary A surface,

$$\dot{W}_{Fp} = \frac{dW_{Fp}}{dt} = \int_A \underline{v}\, p d\underline{A}$$

Where the $pd\underline{A}$ product is the elementary pressure force ($d\underline{F}_p$). When the direction of the flow parallel to surface element vectors and with average velocity magnitude, temperature, density and pressure, the energy flowrates and power of the pressure forces,

$$\frac{(a - v')^2}{2} \rho_0 A a + (T_0 + T') c_V \rho_0 A a - \frac{a^2}{2} \rho_0 A a - T_0 c_V \rho_0 A a = a A p_0 - (a - v') A (p_0 + p')$$

Dividing both sides of the equation by the mass flowrate, after performing the simplifications and neglecting the small terms in the second order

$$-a v' + T' c_V = \frac{p_0}{\rho_0} - \frac{p_0 + p'}{\rho_0 + \rho'}$$

Bringing the right side to the same denominator and introducing the simplification $\rho_0+\rho'\approx\rho_0$ in the denominator,

$$-av' + T'c_V = \frac{p_0\rho'}{\rho_0^2} - \frac{p'}{\rho_0}$$

In the obtained equation, based on the linear acoustic motion equation, the two terms at the sides of the equation are eliminated, the algebraic shape of the linear acoustic energy equation expressed to the temperature fluctuation variable,

$$T' = \frac{p_0}{c_V\rho_0^2}\rho'$$

State equation for ideal gases:

$$\frac{p}{\rho} = RT$$

Equality is also true for the elementary change of the left and right sides of the term,

$$d\left(\frac{p}{\rho}\right) = d(RT)$$

Rewritten for elementary changes in members,

$$\frac{dp}{\rho} - \frac{p}{\rho^2}d\rho = RdT$$

Based on the empirical finding, the acoustics is the “mechanics of the small changes of large quantities” we approximate that the elementary quantities correspond to the fluctuating (dash index), and the total quantities correspond to the equilibrium (0 index) terms. Based on this, the linear acoustic ideal gas equation of state,

$$\frac{p'}{\rho_0} - \frac{p_0}{\rho_0^2}\rho' = RT'$$

Comments:

- According to the low amplitude nature of sound, a simple linear algebraic relationship between the variables describing the sound field (p’, v’, T’ and ρ’) can be determined.

- The direct advantage of the linear algebraic relationship between the acoustic variables (p’, v’, T’ and ρ’) is that the knowledge of one variable (e.g. based on experimental studies or more complex place- and time-dependent calculations) the other variables can also be determined.

- Three equation (continuity, motion and energy equation) from the linear algebraic equations are direct expressions (a constant times one sound field variable results the other sound field variable).

- Mathematical consequences of linearity, the simple representation (formalism), and the applicability of the linear superposition principle. In acoustics, the linear superposition principle means that for a composition of two or more sound fields, the resulting sound field variable is a simple algebraic sum of variables of the component sound fields (e.g., for a composition of two sound fields, the resulting sound pressure, ${p'}_{1+2} = {p'}_1 + {p'}_2$)

- The physical consequence of linearity is that sound waves existing at the same time and in same place do not interact with each other, do not distort each other (e.g.: the voice of a person speaking while listening to music does not change the melody, and vice versa).

- The linear acoustic equation of motion is not a common equation of motion because it creates a relationship between velocity and pressure (force) rather than acceleration and pressure (force).

- The linear acoustic equation of motion can be used not only for small changes, but also for estimation in the case of large-amplitude disturbances. With its help, in the case of a sudden blockage if the flow in a rigid-walled pipe, the pressure increase as a function of the change in velocity can be estimated (simple Allievi's theory).

- In the linear acoustic motion equation, replace the particle velocity from the linear acoustic continuity equation, and then express the speed of sound from it,

$$a^2 = \frac{p'}{\rho'}$$

Based on this relationship, the speed of sound cannot be quantified numerically, but it can be physically concluded that the speed of sound in a less compressible medium is higher (the same pressure disturbance produces a smaller change in density resulting in a higher quotient and a higher speed of sound).

- The relation suitable for calculating the speed of sound can be derived by substituting the linear acoustic energy equation for the linear acoustic state equation for gases,

$$\frac{p'}{\rho_0} - \frac{p_0}{\rho_0^2}\rho' = R\frac{p_0}{c_V \rho_0^2}\rho' \quad \text{after rearrangement,} \quad \frac{p'}{\rho'} = \frac{p_0}{\rho_0}\left(1 + \frac{R}{c_v}\right) = \frac{p_0}{\rho_0}\left(\frac{c_v}{c_v} + \frac{R}{c_v}\right) = \frac{p_0}{\rho_0}\frac{c_p}{c_v} = \kappa\frac{p_0}{\rho_0} = \kappa R T_0$$

During the manipulation let take into consideration, $R=c_p-c_v$, and $\kappa=c_p/c_v$. Using the expression of sound speed, and the state equation of perfect gases for the equilibrium variables, the expression of speed of sound is,

$$a = \sqrt{\kappa R T_0}$$

- Based on the relation, it can be concluded that in the case of isentropic change of state in gaseous medium, the speed of sound (a) depends only on the material quality of the gas (adiabatic coefficient (κ) and specific gas constant (R)) and the equilibrium temperature (T_0). Using the relation, for example, in 20 degree Celsius air, the value of the speed of sound, rounded to the integer, is 343 m/s (κ= 1.4; R_{lev} = 287 J/kgK and T_0 = 293 K). The use of our linear model is greatly simplified by the fact that the speed of sound does not depend on frequency and intensity. Due to the frequency independence of sound propagation, no colour dispersion occurs in airborne sounds (colour dispersion: waveform modification due to different propagation speeds of different frequency wave components).

- In later derivations, to understand the simplifications, it is instructive to determine the relations between the numerical magnitudes of the acoustic variables. For this, the values of particle velocity, temperature and density fluctuations are expressed as a function of sound pressure. The prominent role of sound pressure can be attributed to the fact that the sound pressure is the only acoustic variable that can easily, experimentally determine (microphone measurements). Special apparatus can be used to measure particle velocity, but no experimental method is available to determine the temperature and density fluctuations in sound field. After the transformation of the derived relationships, the v', T' and ρ' variables and their magnitude as a function of sound pressure (if a= 340 m/s, ρ_0= 1.2 kg/m^3 and c_p= 1000 J/kgK), are,

$$p' > v' = \frac{p'}{\rho_0 a} \approx \frac{p'}{400} > T' = \frac{p'}{c_p \rho_0} \approx \frac{p'}{1200} > \rho' = \frac{p'}{a^2} \approx \frac{p'}{115600}$$

We have already mentioned that the hearing threshold at 1kHz is $2\cdot10^{-5}$Pa effective sound pressure, the corresponding particle velocity is $5\cdot10^{-8}$ m/s, the temperature fluctuation is $1.67\cdot10^{-8}$ K, and the density fluctuation is $1.73\cdot10^{-10}$ kg/m^3, very small values in engineering approach. Among the variables, the value of ρ' is the smallest, so it is neglected many times during the derivation, despite the fact that the compressibility of the medium is an essential condition for the formation of the sound field.

- Till the amplitude of the sound field variables is small, the accuracy of the linear algebra model is adequate. At high amplitudes (e.g. during sound generation, in the case of resonance) the accuracy of the linear model decreases rapidly. Another application limitation is that we did not take into account the losses during sound propagation in the mathematical model. Outside the source region (usually a few meters away from the sound source), within the sound attenuation range (taking into account the frequency range important for engineering noise control within a few times 100m from the sound source), avoiding resonant behaviour, the linear model works well. In technical practice, for most acoustic and noise control problems, these conditions are met, so the system of direct algebraic relationships between sound field variables can be widely used in studies related to sound fields.

2.3. Test questions and solved problems

T.Q.1. Derive the linear relationships between sound pressure, particle velocity, density and temperature fluctuations!

T.Q.2. List and analyse the mathematical and physical consequences of a linear relationship between variables describing sound field!

T.Q.3. Starting from the basic equations of fluid dynamics, derive and analyse the relationship of the speed of sound as a function of the equilibrium temperature of the medium, assuming an isentropic change of state!

S.P.1. Determine the maximum value of the pressure, density and temperature fluctuations (p'_{max}, ρ'_{max} és T'_{max}) of a plane wave propagating freely in air, if the maximum value of the particle velocity (v'_{max}) is 0.015 m / s. The air temperature (t) is 25 °C, pressure (p) 1bar, adiabatic coefficient (κ) 1.4, specific gas constant (R) 287J/kgK, specific heat (c_p) 1000J/kgK at constant pressure.

$$a = \sqrt{\kappa R T_0} = \sqrt{1.4 \cdot 287 \cdot (273+25)} \approx 346 \ m/s$$

$$p_0/\rho_0 = RT_0 \ , \ \rho_0 = p_0/RT_0 = 10^5/287 \cdot (273+25) \approx 1.17 \ \ kg/m^3$$

$$c_p/c_v = \kappa \ , \ c_v = c_p/\kappa = 1000/1.4 \approx 714{,}3 \ \ J/kgK$$

$$p'_{max} = \rho_0 a v'_{max} \approx 1.17 \cdot 346 \cdot 0.015 \approx 6.1 \ Pa$$

$$v'_{max} = \rho'_{max} a/\rho_0 \ , \ \rho'_{max} = v'_{max}\rho_0/a \approx 0.15 \cdot 1.17/346 \approx 5.1 \cdot 10^{-5} \ kg/m^3$$

$$T'_{max} = \rho'_{max} p_0/c_V \rho_0^2 \approx 5.1 \cdot 10^{-5} \cdot 10^5/714.3 \cdot 1{,}17^2 \approx 0.0052 \ \ K$$
