Technical Acoustics and Noise Control *(lecture notes for self-learning)*

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Lecture 5.

Content:

5.1. Important composition of harmonic waves

In the next section, using our knowledge of wave acoustics, we present two theoretically and practically important composition of harmonic waves, the standing wave and the beat. To expand our knowledge, we derive a mathematical model of complex waves, and then, by examining the solution, we further deepen our knowledge of the phenomena.

Standing wave:

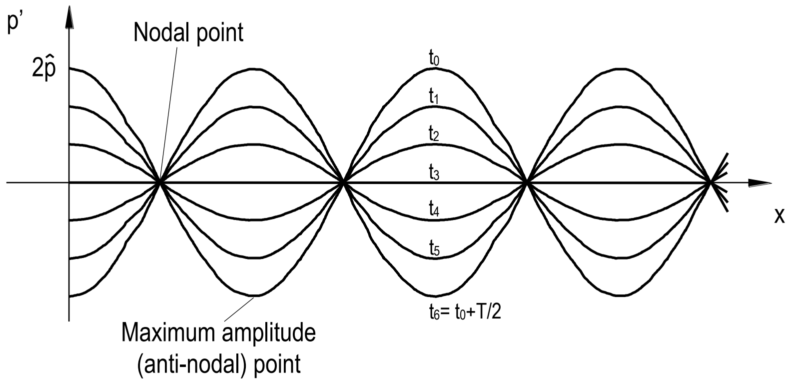
The composition of two harmonic waves of the same frequency, amplitude and propagation velocity, traveling in opposite directions creates standing wave. The wave function of the standing wave is derived using the linear superposition principle. The resulting sound pressure of the two sound fields can be determined by the simple algebraic sum of the sound pressures of the component wave. The final result of the derivation can be obtained using trigonometric identities for the decomposition of the cosine of the angle sum and difference,

$$\hat{p}_1 = \hat{p}_2 = \hat{p} \;,\quad \omega_1 = \omega_2 = \omega \;,\quad k_1 = k_2 = k \;,\quad |a_1| = |a_2| \;,\quad a_1 \longrightarrow_{+x} \text{ and } a_{2\,-x}\longleftarrow$$

$$p'_{ah}(x,t) = p'_1(x,t) + p'_2(x,t) = \hat{p}_1 \cos(\omega_1 t - k_1 x) + \hat{p}_2 \cos(\omega_2 t + k_2 x) =$$

$$= \hat{p}\left(\cos(\omega t - kx) + \cos(\omega t + kx)\right) = 2\hat{p}\cos(\omega t)\cos(kx)$$

The graph of the standing wave within a half-period time range at different times is shown along the x direction in the following figure.



The sound pressure variation of standing wave versus distance at different times

Comments:

- Similar to the enclosed space solution of the wave equation, in the case of a standing wave the place and time variables are separated in the argument of the solution function, i.e. the propagating nature of the wave disappears.

- In the standing wave, the interaction of the opposing components results in certain places extinction of the pressure fluctuation that is constant in time or amplification between them. Nodes form at the extinction places and maximum amplitude variation at the anti-node places.

- An important difference between the enclosed space solution of the wave equation and the standing wave is that in the enclosed space solution the amplitude depends on the initial condition and can be practically arbitrary, while in the standing wave it can be at most the sum of the amplitudes of the two interfering components (traveling to right and left).

- The standing wave is a one-dimensional sound interference phenomenon.

- The formation of a standing wave can also be understood graphically. Lay the graphs of two identical sine functions exactly on top of each other, and then draw the two curves on top of each other with velocities of the same magnitude but of opposite orientation. From a steady observation point, where there were zero values in the original state, values with the same magnitude but opposite sign are generated after the displacement, their sum is constantly zero. Offset by a quarter of a period (90 degrees) the summation of the signed sections will result a periodic fluctuation between the minimum and maximum.

- In practice, a standing wave may occur when a poor harmonic sound wave (poor tone) falling perpendicular to a wall is reflected. A plastered brick or concrete wall reflects up to 95% of the incident sound energy, depending on the frequency. Thus, the amplitudes are approximately the same, the reflection does not affect the frequency and the absolute value of the speed of sound, so that an incident and reflected wave creates a standing wave.

- Take care of standing waves is also important from a metrological point of view. In the outdoor places partially bordered by sound-reflecting walls, between the sound source and the sound-reflecting surface an unexpected sound pressure variation can appear as a function of location (e.g.: moving away from the sound source, instead of decreasing, locally the sound pressure will increase).

Beat:

The composition of two harmonic waves of the same amplitude and propagation speed traveling in one direction, with slightly different frequencies, creates beat. To determine the wave function of the beat, we also start from the principle of linear superposition,

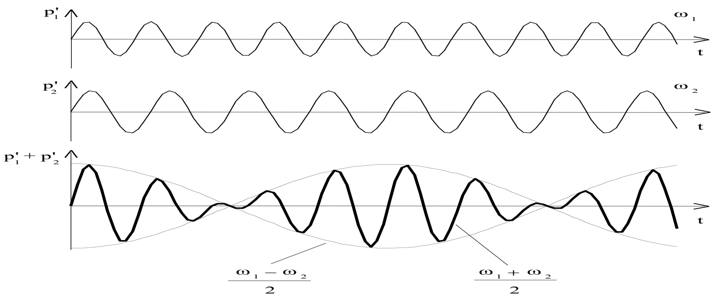
$$\hat{p}_1 = \hat{p}_2 = \hat{p} \;\;, \quad a_1 = a_2 \;\;, \quad \omega_1 > \omega_2 \;\text{, but}\; \omega_2 \gg \omega_1 - \omega_2 \quad , \; k_1 > k_2 \;\text{, but}\; k_2 \gg k_1 - k_2$$

$$p'_l(x,t) = p'_1(x,t) + p'_2(x,t) = \hat{p}_1\, cos(\omega_1 t - k_1 x) + \hat{p}_2\, cos(\omega_2 t - k_2 x) =$$

$$= \hat{p}\,\left(cos(\omega_1 t - k_1 x) + cos(\omega_2 t - k_2 x)\right) =$$

$$= 2\hat{p}\, cos\left(\frac{\omega_1+\omega_2}{2}t - \frac{k_1+k_2}{2}x\right) cos\left(\frac{\omega_1-\omega_2}{2}t - \frac{k_1-k_2}{2}x\right)$$

The final result obtained by using the identity of the sum of the cosines of the angles, the beat wave function, is not simpler than the initial expression. But by forming appropriate group of the variables, it makes more understandable the physically relevant properties of beat.



The sound pressure time history of the component waves and the resulted beat at x = 0m position

The previous figure shows a time-dependence of the beat wave function at the x= 0m fixed position, using graphical summary of the component sound wave functions.

Comments:

- Due to the composition of the sound waves, the wave function has been greatly transformed (instead of the sum of cosines, they have been multiplied), but the place and time variables can be found together in the argument of both multiplied cosines, so like the components sound waves, the beat remains a propagating wave.

- The graphical summary shown in the previous figure shows the physical explanation for the occurrence of beat. Initially, the waves starting from the same phase mutually amplifying each other, due to the small difference in angular velocities, over a longer period of time compared to the base period, the phases will change reverse and the waves cancel each other and then this is repeated.

- In the case of beat, the sound observed at a given point is a pure tone, its frequency is the simple algebraic mean of the component frequencies, and this base sound wave is modulated by half the difference of the frequencies of the two components. During modulation, our hearing perceives amplifications and attenuations (it does not differentiate between the positive and negative halves of the envelope), so the subjectively perceptible modulation frequency is exactly twice the calculated value.

- In the case of beat, the sound energy propagates in groups, in the widening sections of the modulation period.

- In practice beat occurs for example, when the operating points of two periodic behaviour noise source (e.g.: ventilation fans) of the same type slightly different to each other. Due to the same type, the noise generation of the equipment is basically the same, but the small difference between the operating points (e.g.: the throttle of one of the fans is higher) results slightly different rotational speeds (the frequency of the generated poor sound component is proportional to the rotational speed) and thus causes slightly different radiated sound frequency, that creates beat.

- The subjective judgment of poor sounds is wrong. Periodic modulation further degrades our subjective opinion, so beat should be avoided from a noise protection point of view. Conversely, and taking advantage of its attention-grabbing, disturbing nature, the periodically modulated sound can be used as an emergency warning sound, e.g., an ambulance car sound signal, or a civil defence (anti-aircraft) siren.

5.2. Model testing of sound fields and acoustic similarity

A model is created when examining the task in its original size is physically not possible, dangerous, expensive, or takes too long. Model can be mathematical or physical. The most important properties of the mathematical model are described in the earlier "Irregular mathematic introduction" section. During physical modeling, the properties of the original phenomenon are determined by experimental methods and measurements. Experimental modelling occurs when there is no mathematical model for the physical problem or a mathematical description exists, but the accuracy of the solution is unclear and needs to be verified ("validation"). The physical model is a simplified version of the original phenomenon, which is similar to the original in terms of the studied properties, but may differ from it in other not important aspects. Experimental models can be homologous or analogous in nature. In homologous modelling, the physical phenomenon used in the experimental study is the same as the original phenomenon (e.g., studying the flow around a car body in a wind tunnel on a smaller scale model). There are cases where the original physical phenomenon is difficult to measure, such as water leakage in porous soil. In this case, the phenomenon is examined with a physical phenomenon different from the original, in which the shape of the functions (with different variables) describing the model physical phenomena is the same as the original phenomenon, but the measurement can be easily implemented. Such a study is called analogous modelling. Analogous modelling, for example, is the study of water leakage in sand with an electrical potential analogy. In acoustics, usually we apply homologous modelling, so the original sound field investigated by a smaller test sound field experiments.

In model testing, it is essential that the original phenomenon and the model phenomenon behave similarly in terms of the selected variable. For example, if in the original size of a concert hall, the sound pressure distribution shows an undesired increase in a certain part of the space, it should also be detectable in the tested low scale model sound field. Similarity can be determined independently by a mathematically and physically based condition. In the case of the mathematically based condition, the original and the model phenomena are similar if the differential equations for phenomena written by dimensionless variables and the related dimensionless boundary and initial value conditions are equal to each other. In the case of homologous modelling, the form of the governing equations are identical, so the dimensionless coefficients of the dimensionless differential equations and the dimensionless boundary and initial value conditions are required for similarity. In the case of the physics-based condition, the quotient of the physical variables determining the phenomenon is constant for both the original and the model phenomena. The quotient of determining variables is, for example in fluid mechanics, the quotient of the inertia force and the viscous force (Reynolds number) in the case of viscous fluid motion, or the quotient of the ordered and disordered kinetic energies (Mach number) in the case of a compressible fluid flow.

In the modelling of sound fields, we start from the homogeneous acoustic wave equation in the mathematically based determination of the similarity condition,

$$\frac{1}{a^2}\frac{\partial^2 p'}{\partial t^2}-\frac{\partial^2 p'}{\partial x^2}=0$$

To create dimensionless variables in the equation, multiply both sides of the equation by the square of the characteristic size of the sound field (L_0) and divide by characteristic reference sound pressure of the sound field (p'_0). Then, to the left of the equation, multiply the numerator and denominator of the first term to the square of the characteristic time of the sound field (T_0). The dimensionless wave equation,

$$\frac{L_0^2}{a^2T_0^2}\frac{\partial^2\left(\frac{p'}{p'_0}\right)}{\partial\left(\frac{t^2}{T_0^2}\right)}-\frac{\partial^2\left(\frac{p'}{p'_0}\right)}{\partial\left(\frac{x^2}{L_0^2}\right)}=0$$

Due to homologous modelling, the dimensionless governing equations of the original and model sound spaces are identical in shape. Thus, the condition for the similarity of the sound fields in the original and model equations on the left is the identity of the coefficients in the first term. If the quotient is constant, necessarily its square root is also constant, and let us also focus our studies to harmonic waves, where the characteristic time of the sound field is the time of period (T_0). The product of the time of period and the speed of sound is the wavelength (λ), which is the condition for the similarity of two sound fields, the Helmholtz number (He),

$$He=\frac{L_0}{aT_0}=\frac{L_0}{\lambda_0}$$

the identity of the ratio of the characteristic geometric size and wavelength of the sound field.

Comments:

- In acoustics, to reduce costs, models are usually smaller than the original size. The similarity condition is a direct proportional relationship between the characteristic size and the wavelength, and an inverse proportional relationship between the characteristic size and the frequency, so the smaller the size, means higher frequency. In mechanical engineering noise control and in room acoustics, the upper value of the frequency range of interest at the original size can reach several kHz, even 10 kHz. Realistically 1:10, but often even smaller models, the expected upper value of the frequency range of the test microphones is well above the audible range, in the tens of kHz range. As a result, the sensitivity of the microphones decreases and the cost of acquiring the instrument increases.

- Another problem is the increasing dissipative losses during sound propagation due to the high sound frequency used in model testing. The attenuation of sounds due to dissipation in air up to a frequency of 1… 2 kHz at a distance of a few 10 m is practically negligible (a few percent of the incident sound power), in the range above 10 kHz the attenuation due to dissipation becomes very significant and cannot be neglected. Dissipative losses during sound propagation can cause model test inaccuracies.

- Let's not forget the flow nature of airborne sounds. Based on this, the similarity numbers determined for the flows can also be used in acoustics with a suitable choice. The flow nature of sound is characterized by its time-varying nature, compressibility, and small amplitudes. The first two characteristics can be assigned the Strouhal- and Mach-number flow similarity numbers, respectively. The similarity calculations are scale rules, so the product of the two similarity numbers must be taken to determine the resultant similarity number,

$$Sh\,Ma = \frac{f_0 L_0}{v_0}\,\frac{v_0}{a_0} = \frac{L_0}{T_0 a_0} = \frac{L_0}{\lambda_0} = He$$

The result is the same, the Helmholtz number. Such a derivation of the Helmholtz number is instructive in that it can be applied in other cases as well. If we do not know the condition of similarity for a complex phenomenon, but we do know the condition of similarity for the component phenomenon, in some cases the condition for similarity of a complex phenomenon is the product of the similarity numbers for the component phenomena.

5.2. Test questions and solved problems

T.Q.1. What is a standing wave, derive the expression of sound pressure as a function of the position and time for any standing wave, and explain its practical importance!

S.P.1. A harmonic wave of 0.001 sec time of period and 0.5 Pa sound pressure amplitude perfectly reverberated from a perpendicular positioned, plane surface. Name the emerging acoustic phenomenon, calculate the characteristic frequency, the distance between two nodal points and the minimum and maximum sound pressure amplitude of the resultant wave! The air temperature is 10°C.

Solution:

The name of this superposition is standing wave.

The frequency is f= 1/T= 1kHz,

The distance between two neighbouring nodal point is the half wave length,

The speed of sound: $a = \sqrt{\kappa R T_0} = \sqrt{1.4 \cdot 287 \cdot (273 + 10)} \approx 337.2\ m/s$

The wave length: $\lambda = aT \approx 337.2 \cdot 0.001 \approx 0.3372\ m$

The distance between two nodal points: $\lambda/2 \approx 0.3372/2 \approx 0.1686\ m$

The resultant sound pressure amplitude in the nodal (minimum) point is 0 Pa.

The resultant sound pressure amplitude in the nodal point is 0Pa, and in the anti-nodal (maximum) point is the simple sum of the component amplitudes, 1Pa.
T.Q.2. What is beat, derive the expression of sound pressure variation as a function of the position and time for any beat, and explain its practical importance!

T.Q.3. What does mean the similarity of two sound fields and give the condition for the similarity of sound fields!

S.P.2. By examining a 1:10 scale model of a theatre hall, we intend to determine the properties of the sound fields formed in the original hall as a result of pure sound excitation at 250, 500 and 1000 Hz. Determine the frequencies of the sound required for the model tests, if in both the original and the model sound space the medium is air in the technical normal state (p_0= 1bar, t_0= 20˚C)!

Solution:

$He_m=He_0$, $l_m/\lambda_m= l_0/\lambda_0$, $l_m f_m/a_m= l_0 f_0/a_0$,   but ($a_m= a_0$)   so, $f_m= f_0\, l_0/l_m =10\cdot f_0$

The test frequencies: 2,5k, 5k and 10kHz
