Technical Acoustics and Noise Control *(lecture notes for self-learning)*

Dr. Gábor KOSCSÓ titular associate professor (BME Department of Fluid Mechanics)

Lecture 6.

Content:

6.1. Acoustic resonators (lecture notes)

Resonator is a vibrating system exciting on its natural frequency. Vibrating system is a physical system that can create vibration. The vibration is a periodic behaviour around the equilibrium state of the system. The periodic behaviour means, that the variables, mathematically describe the system, alternating around the mean value. To create vibration inertia and restoring force is required. For vibrating system, a good and simple example is the mass and spring one degree of freedom mechanic vibrating system, where the mass is the inertia and the spring will create the restoring force.

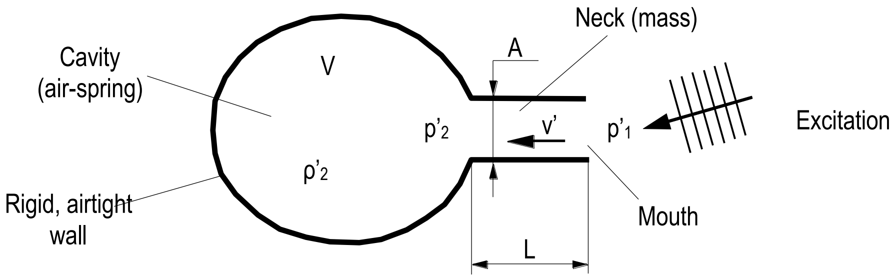
The vibrating systems can classify in concentrated and distributed parametric types. At concentrated parametric vibrating system the physical features, responsible to form vibration can concentrate to a small section of the space. In this case to describe the motion the time variable is enough, the equations will be ordinary differential equations. The mass and spring one degree of freedom mechanic vibrating system is a typical concentrated parametric vibrating system. At distributed parametric vibrating system the physical features, responsible to form vibration and the occurrence itself will be distributed is space. In this case to describe the vibration space and time variable required as well, so the governing equations will be partial differential equations. The vibrating guitar string, the spiral spring or an air cylinder closed in a pipe are distributed parametric vibrating systems.

Taking into account internal and external effects, vibrations can classify as free, attenuated and excited types. At free vibration the system after the initial perturbation works itself, and the absence of losses the periodic motion will take continuously in time without any change. In real cases, in mechanics the motions are escorted by forces that are oppose the motion and creating losses. Concerning vibrations, the name of this process called attenuation. For example the attenuation of the one degree of freedom mechanic vibrating system caused by the internal friction of the spring, the air drag of the moving mass and the not perfectly rigid connection of the other end of the spring (this latter is not a dissipative loss). A vibration escorted by losses called attenuated vibration. The amplitude of an attenuated vibration will continuously decrease in time, until it will stop. The excited or forced vibration will governed by a continuous, external effect.

A special case among the excited vibrations are the resonance. The resonance has elevated importance in mechanical engineering. In the case of resonance the natural frequency of the system is equal the frequency of the excitation. The constant phase behaviour of the system and excitation will result, that work taken by the excitation, the vibrating system completely admits and conserves. The resonators can characterise with small excitation amplitude and big answer amplitude. As a result of a continuous excitation, the amplitude in a not attenuated vibrating system will increase continuously in time. In attenuated system the amplitude will grow up until the work taken by the excitation during one period and the losses caused by the attenuation during one period will be equal to each other. In a steel structure the losses are very small, so this equilibrium will take place at great amplitudes. In mechanical engineering systems the disordered big amplitude operation occasionally can effect malfunction or accident of the machine, so generally in mechanical engineering we try to avoid the resonance. In acoustics the rating of resonance is not clear. In building acoustics, mechanical engineering noise control and room acoustics the resonances are mostly undesired, but for example at the ultrasonic cosmetic technologies or the music instruments the resonance has an organic part in the operation.

The Helmholtz-resonator

The Helmholtz-resonator or cavity resonator looks like an „Unicum" bottle. The important parts are the cavity, neck and mouth. To create a vibration in the resonator let's make a sound pressure impulse in the vicinity of the mouth. Because of the small positive pressure difference, an elementary fluid flow will be generated in the neck. The consequence of the small volumetric fluid transport, the pressure will be elevated in the cavity. After the sudden end of the pressure impulse, the ambient pressure of the resonator turns to the original equilibrium atmospheric value, so the air motion in the neck slowly decelerates, and stops for a moment. In this time the internal pressure in the cavity starts a motion again, but directed outside from the cavity. Due to the fluid transport, after a few time the pressure inside and outside of the cavity will be equal, but the inertia of the air, inside the neck does not allow to stop the motion. In absence of losses, the f_0 natural frequency periodic motion will continue till arbitrary time. The main principle of this process is identical to the operation of the mass and spring one degree of freedom mechanical vibrating system. The acoustic and mechanical vibrating systems are analogous to each other. The Helmholtz resonator is a concentric parametric, one degree of freedom acoustic vibrating system, where air closed in the neck is the mass, and the air inside the cavity is the spring. If change the pressure impulse excitation to a continuous pure tone sound of f_0 natural frequency, the system will resonate.



The schematic drawing of the Helmholtz-resonator

Our goal is to describe the processes inside the Helmholtz-resonator, how effect the external pressure excitation the motion inside the neck. The concentrated parametric system can mathematically describe with ordinary differential equation. The simplifications outside the resonant frequency region are same, applied in linear acoustics. During resonance, the amplitudes are increased, so the non-viscose and adiabatic conditions are not satisfied. Despite of these effects, for easier mathematic derivations, we will apply on the complete frequency range the simplifications, than was in linear acoustics. The consequences of this we should take care later during the use of the solution.

Continuity equation: The mass conservation for the resonator is, the mass change of the air inside the cavity per unite time must be equal with the mass flow rate of the air inside the neck,

$$\left(\frac{dm}{dt}\right)_{cavity} = q_{m\,neck} \qquad \frac{d}{dt}(V\rho_2) = \rho_{neck}Av'$$

If the wall of the resonator is rigid, the pressure difference cannot change cavity volume (V), and decompose the acoustic variables to equilibrium and time variant components,

$$V\frac{d}{dt}(\rho_0 + \rho'_2) = (\rho_0 + \rho'_{neck})Av'$$

The time derivatives of the equilibrium values are zero, and suppose, $\rho_0+\rho'_{neck} \approx \rho_0$, the continuity equation for the Helmholtz-resonator is,

$$V\frac{d\rho'_2}{dt} = \rho_0 Av'$$

The equation of motion: Suppose, the length of the neck (L) is much smaller, than the wave length of the exciting sound wave, and the air, inside the neck, will move as a rigid body. The resultant force acting on the air at both side of the neck will equal with the product of the acceleration and mass of the air inside the neck,

$$\frac{dv'}{dt}m_{neck} = \sum F_p$$

$$\frac{dv'}{dt}LA\rho_0 = (p_0 + p'_1)A - (p_0 + p'_2)A$$

The equation of motion for the Helmholtz-resonator,

$$\frac{dv'}{dt}L\rho_0 = p'_1 - p'_2$$

From the linear algebraic acoustic model, the seed of sound as a function of p' and ρ',

$$\frac{p'}{\rho'} = a^2 = \kappa R T_0$$

The excitation: The pressure fluctuation at the mouth of the resonator is,

$$p'_1 = \hat{p}_1 \cos \omega t$$

After some mathematic operation,

$$\frac{dp'_2}{dt} = \frac{a^2 \rho_0 A}{V} v'$$

$$\frac{d^2 v'}{dt^2} L\rho_0 = \frac{dp'_1}{dt} - \frac{dp'_2}{dt} = -\omega \hat{p}_1 \sin \omega t - \frac{a^2 \rho_0 A}{V} v'$$

The governing equation of the Helmholtz-resonator,

$$\frac{d^2 v'}{dt^2} + \frac{a^2 A}{LV} v' = -\frac{\omega \hat{p}_1}{L\rho_0} \sin \omega t$$

And the solution of the second order, inhomogeneous, ordinary differential equation, the particle velocity inside the neck as a function of time,

$$v'(t) = \hat{v} \sin(\omega t)$$

Where,

$$\hat{v} = \frac{-\frac{\omega \hat{p}_1}{L\rho_0}}{\frac{a^2 A}{LV} - \omega^2}$$

To look better the phase difference between the excitation and the particle velocity let's change sinus to the cosine as it was at the excitation,

$$v'(t) = \hat{v} \cos\left(\omega t + \frac{\pi}{2}\right)$$

Comments

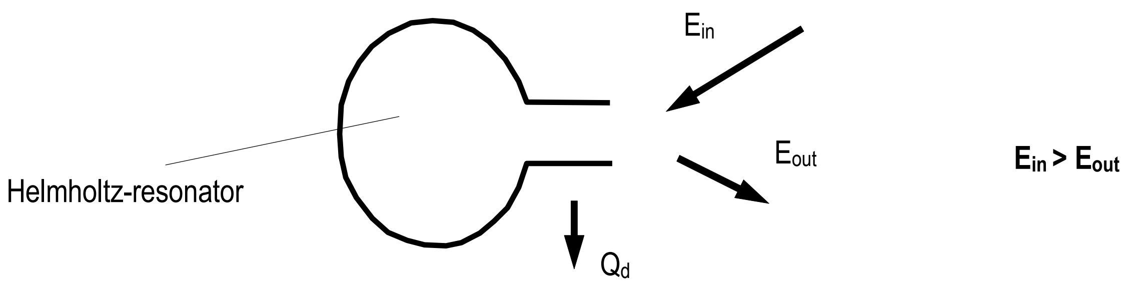
- As a result of the ω angular frequency excitation, the angular frequency of the Helmholtz-resonator vibration will have the same ω value as well. The frequency of the excitation and of the system vibration are the same.
- The phase shift between the particle velocity inside the neck and the excitation is π/2 radian (quarter period). When the magnitude of the particle velocity is maximum, the sound pressure of the excitation is zero, and reverse, like at the mass and spring mechanical vibrating system.
- The particle velocity amplitude effected by the frequency. At small frequencies the particle velocity amplitudes are very small. Increasing the frequency, close to the resonance frequency (ω_r) the particle velocity amplitude will increase seriously,

$$\omega_r = \sqrt{\frac{a^2 A}{LV}}$$

When the excitation frequency tends to resonance frequency the particle velocity amplitude tends to infinite. In reality, it cannot be happen. This is the penalty because of the neglected attenuations effects. Over the resonant frequency, the particle velocity will decrease again.

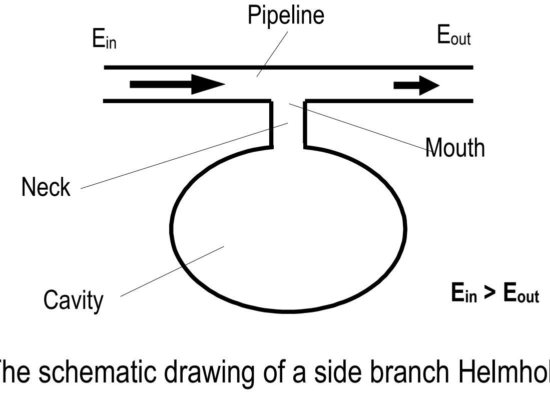
The effects and application of the resonators:

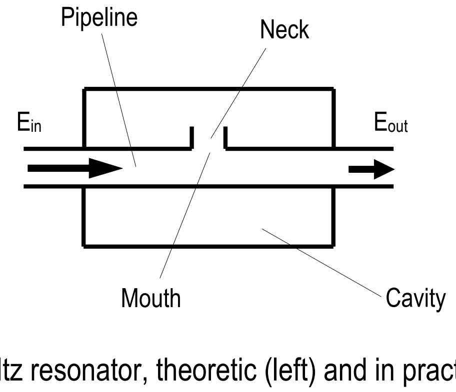
- Attenuation: During the resonant behaviour, because of the dissipative effects, the back radiated acoustic energy will be smaller than the incident one, so the Helmholtz-resonator will behaves like an attenuator.



The attenuation effect of a Helmholtz-resonator

- Destructive interference: A Helmholtz-resonator connected to pipe will form a side branch resonator. The sound, propagates in the tube will excite the resonator. The back scattered pressure perturbation created by the resonator will be out of phase to the original excitation, that results a destructive interference.

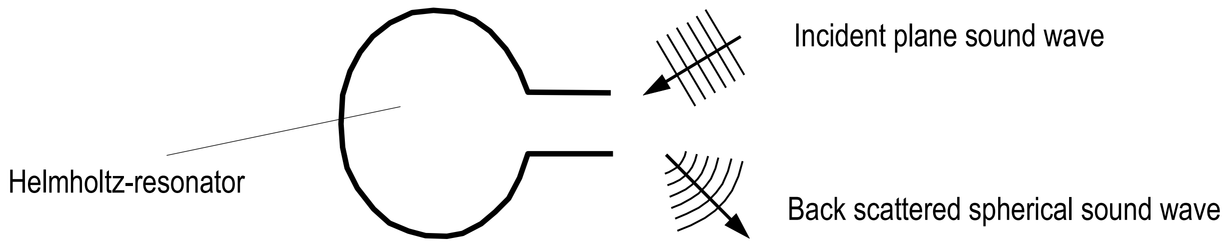




The schematic drawing of a side branch Helmholtz resonator, theoretic (left) and in practise (right)

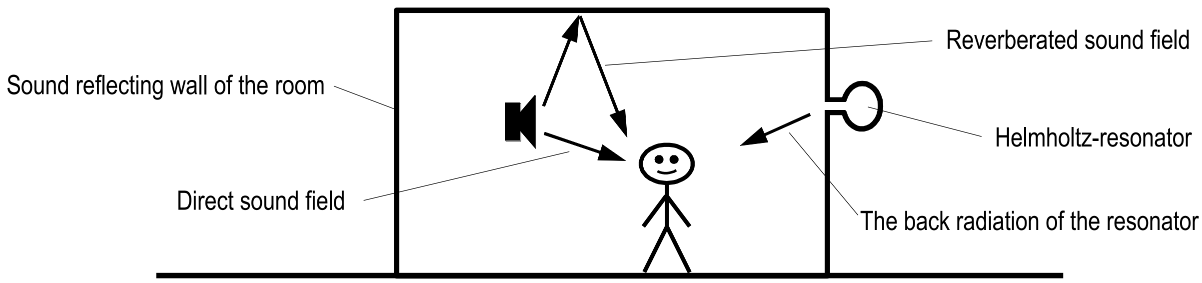
As a consequence of the destructive interference the transmitted sound energy will be smaller than the incident one.

- Scattering: The incident plane wave will be back scattered as spherical wave by the Helmholtz resonator.



The scattering of sound wave during the back radiation of the Helmholtz-resonator

- Reverberation: If the internal surface of a room consists of sound reflecting material, the sound field of the source placed in this room can share into direct and reverberant components. The reverberated sound field is well observable in a large, good sound reflecting surface halls (churches, public bath hall) when creating a short time impulse sound effect (clap the hands). The energy of the reverberated sound field proportional with the volume of the room, and the absorption of the reverberated sound will proportional with the area of the internal surface. In small size rooms the specific surface (the ratio of the surface and volume) is bigger than in large one, so the absorption will be faster and the reverberation time shorter.



The back radiation of the Helmholtz-resonator will increase the reverberation time in a room

A Helmholtz-resonator connected to the room, will excited by the internal sound field. Turning off the sound source in a small room, the reverberant sound field will decay rapidly, but the excited resonator, depending on its attenuation, will back radiate sound energy longer and increase the reverberation time. Longer reverberation sounds like larger room, so the cavity resonator built in the wall, can magnify the acoustic space of a room.

8.2. Test questions and solved problems

T.Q.1. Based on the equation of fluid mechanics derive the governing equation for Helmholtz resonator. List the simplifications, explain the neglected terms in detail, and put down the applications of the resonator. Make a schematic drawing to the answer!
