Turbulence modelling I.

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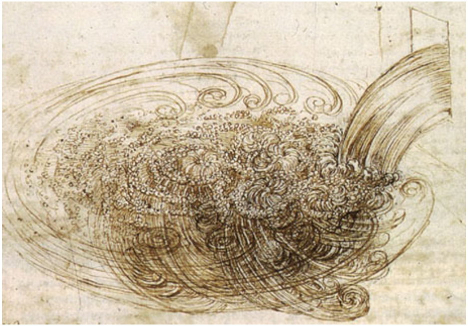
History - Leonardo da Vinci, ca. 1500

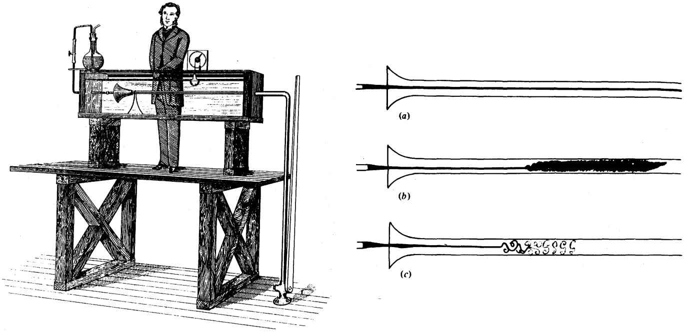
Leonardo da Vinci (translation: Piomelli in Lumley, J.L., 1997):

"Observe the motion of the surface of the water, which resembles that of hair, which has two motions, of which one is caused by the weight of the hair, the other by the direction of the curls; thus the water has eddying motions, one part of which is due to the principal current, the other to random and reverse motion"



Art - Vincent van Gogh : Starry night, 1889

Science - Reynolds experiment, 1883



Science - Quotations

Horace Lamb, 1932

„I am an old man now, and when I die and go to heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the turbulent motion of fluids. And about the former I am rather optimistic."

Peter Bradshaw, 1994

„Turbulence was probably invented by the Devil on the seventh day of Creation when the Good Lord wasn't looking."

Introduction

Why to deal with turbulence in a CFD course?

- Most of the equations considered in CFD are model equations
- Turbulence is a phenomena which is present in $\approx 95\%$ of CFD applications
- Turbulence can only be very rarely simulated and usually has to be modelled
- Basics of turbulence are required for the use of the models

Our limitations, simplifications

Following effects are not considered:

- density variation ($\rho = const.$)
  - Shock wave and turbulence interaction excluded
  - Buoyancy effects on turbulence not treated
- viscosity variation ($\nu = const.$)
- effect of body forces ($g_i = 0$)
  - Except free surface flows, gravity has no effect, can be merged in the pressure

Definition

Precise definition?

- No definition exists for turbulence till now
- Stability, chaos theory are the candidate disciplines to provide a definition
  - But the describing PDE's are much more complicated to treat than an ODE
- Last unsolved problem of classical physics ('Is it possible to make a theoretical model to describe the statistics of a turbulent flow?')
- Engineers still can deal with turbulence

Properties

Instead of a definition

- Properties of turbulent flows can be summarized
- These characteristics can be used:
    - Distinguish between laminar (even unsteady) and turbulent flow
    - See the ways for the investigation of turbulence
    - See the engineering importance of turbulence

High Reynolds number

Reynolds number

- $Re = \frac{UL}{\nu} = \frac{F_{inertial}}{F_{viscous}}$
- high Re number $\longleftrightarrow$ viscous forces are small
- But inviscid flow is not turbulent

Role of Re

- Reynolds number is the bifurcation (stability) parameter of the flow
- $Re_{cr} \approx 2300$ for pipe flows
- $Re > Re_{cr} \Rightarrow$ flow becomes unstable, turbulent

Disordered, chaotic

- Terminology of dynamic systems
- Strong sensitivity on initial (IC) and boundary (BC) conditions
- Statement about the stability' of the flow
- PDE's (partial differential equations) have infinite times more degree of freedom (DoF) than ODE's (ordinary differential equations)
  - Much more difficult to be treated
  - Can be the candidate to give a definition of turbulence
- The tool to explain difference between turbulence and simple' laminar unsteadiness

Continuous spatial spectrum

Spatial spectrum

- Spatial spectrum is analogous to temporal one, defined by Fourier transformation
- Practically periodicity or infinite long domain is more difficult to find
- Visually: Flow features of every (between a bound) size are present

Counter-example

Acoustic waves have spiked spectrum, with sub and super harmonics.

3D phenomena

- Vortex stretching (see e.g. Advanced Fluid Dynamics) is only present in 3D flows.
- In 2D there is no velocity component in the direction of the vorticity to stretch it.
- Responsible for scale reduction
- Responsible to vorticity enhancement

Averaged flow can be 2D

- Unsteady flowfield must be 3D
- The (Reynolds, time) averaged flowfield can be 2D
  - Spanwise fluctuations average to zero, but are required in the creation of streamwise, wall normal fluctuations

Unsteady

Turbulent flow is unsteady, but unsteadiness does not mean turbulence

Stability of the unsteady flow can be different

- In a unsteady laminar pipe flow (e.g. $500 < Re_b(t) < 1000$), the dependency on small perturbations is smooth and continuous
- In a unsteady turbulent pipe flow (e.g. $5000 < Re_b(t) < 5500$), the dependency on small perturbations is strong

Continuum phenomena

- Can be described by the continuum Navier-Stokes (NS) equations
- I.e. molecular phenomena is not involved as it was though 100 years ago

Conclusions

1. Can be simulated by solving the NS equations (Direct Numerical Simulation = DNS)
2. A smallest scale of turbulence exists, which is usually remarkable bigger than the molecular scales
3. The are cases, where molecular effects are important (re-entry capsule)
4. Turbulence is not fed from molecular resonations, but is a property (stability type) of the solution of the NS

Dissipative

Dissipative

- Def: Conversion of mechanical (kinetic energy) to heat (raise the temperature)
- It is always present in turbulent flows
- It happens at small scales of turbulence, where viscous forces are important compared to inertia
- It is a remarkable difference compared to wave motion, where dissipation is not of primary importance

Vortical

Turbulent flows are always vortical

- Vortex stretching is responsible for scale reduction
- Dissipation is active on the smallest scale

Diffusive

Diffusive property, the engineering consequence

- In the average turbulence usually increase transfers
  - E.g. friction factors are increased (e.g. λ)
  - Nusselt number is increased
- In the average turbulence usually increase transfer coefficients
  - Turbulent viscosity (momentum transfer) is increased
  - Turbulent heat conduction coefficient is increased
  - Turbulent diffusion coefficients are increased

Has history, flow dependent, THE TURBULENCE does not exist

As formulated in the *last unsolved problem of classical physics* no general rule of the turbulence could be developed till now.

No universality of turbulence has been discovered

- Turbulent flows can be of different type, e.g.:
    - It can be boundary condition dependent
    - It depends on upstream condition (spatial history)
    - It depends on temporal history

Summarized properties

- High Reynolds number
- Disordered, caotic
- Continous spatial spectrum
- Spatial and temporal phenomena (4D)
- Continuum phenomena (not molecular)
- Vortical
- Dissipative, Diffusive
- Has history

Notations

Directions

- x, u: Streamwise direction
- y, v: Wall normal direction, points to the highest gradient
- z, w: Bi normal to x, y, spanwise direction

Index notation

$x = x_1,\ y = x_2,\ z = x_3\ u = u_1,\ v = u_2,\ w = u_3$

Partial derivatives

$$\partial_j \stackrel{\text{def}}{=} \frac{\partial}{\partial x_j}\ \partial_t \stackrel{\text{def}}{=} \frac{\partial}{\partial t}$$

Summation convention

Summation is carried out for double indices for the three spatial directions.

Very basic example

Scalar product:

$$a_i b_i \stackrel{\text{def}}{=} \sum_{i=1}^{3} a_i b_i \tag{1}$$

NS as example

Continuity eq. in vectorial form and with short notation

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho \mathrm{v}) = 0 \tag{2}$$

$$\partial_t \rho + \partial_i u_i = 0 \tag{3}$$

Momentum eq. in vectorial form and with short notation

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = \boldsymbol{g} - \frac{1}{\rho} \nabla p + \nu \left[\nabla^2 \boldsymbol{u} + \frac{1}{3} \nabla \left(\nabla \cdot \boldsymbol{u} \right) \right] \tag{4}$$

$$\partial_t u_i + u_j \partial_j u_i = g_i - \frac{1}{\rho} \partial_i p + \nu \left[\partial_j \partial_j u_i + \frac{1}{3} \partial_i \left(\partial_j u_j \right) \right] \tag{5}$$

Statistical description

The simple' approach

Turbulent flow can be characterised my its time average and the fluctuation compared to it

Problems of this approach

- How long should be the time average?
- How to distinguish between unsteadiness and turbulence?

Flow examples

- Turbulent pipe flow having ($Re >> 2300$), driven by a piston pump (sinusoidal unsteadiness)
- Von Kármán vortex street around a cylinder of $Re = 10^5$, where the vortices are shedding with the frequency of $St = 0.2$

Difficult to distinguish between turbulence and unsteadiness

Ensemble average

Why to treat deterministic process by statistics?

- NS equations are deterministic (at least we believe, not proven generally)
- I.e. the solution is fully given by IC's and BC's
- Statistical description is useful because of the chaotic behaviour
  - The high sensitivity to the BC's and IC's
  - Possible to treat result of similar set of BC's and IC's statistically

Statistics

Solution as a statistical variable

$$\varphi = \varphi(x, y, z, t, i) \qquad (6)$$

Index i corresponds to different but similar BC's and IC's

Density function

- Shows the 'probability' of a value of φ.

$$f(\varphi) \qquad (7)$$

- It is normed:

$$\int_{-\infty}^{\infty} f(\varphi) \quad d\varphi = 1 \qquad (8)$$

Mean value

Expected value

$$\overline{\varphi(x,y,z,t)} = \int_{-\infty}^{\infty} \varphi(x,y,z,t) \quad f\Big(\varphi(x,y,z,t)\Big) \quad d\varphi \qquad (9)$$

Average

$$\overline{\varphi(x,y,z,t)} = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \varphi(x,y,z,t,i) \qquad (10)$$

Reynolds averaging

Reynolds decomposition

Since the ensemble averaging is called Reynolds averaging, the decomposition is named also after Reynolds

$$\varphi = \overline{\varphi} + \varphi' \tag{11}$$

Fluctuation

$$\varphi' \stackrel{\text{def}}{=} \varphi - \overline{\varphi} \tag{12}$$

Properties of the averaging

Linearity

$$\overline{a\varphi + b\psi} = a\overline{\varphi} + b\overline{\psi} \tag{13}$$

The Reynolds averaging acts only once

$$\overline{\overline{\varphi}} = \overline{\varphi} \tag{14}$$

Properties of the averaging (contd.)

Average of fluctuations is zero

$$\overline{\varphi'} = 0 \tag{15}$$

Deviation

Deviation

- First characteristics of the fluctuations
- $$\sigma_\varphi = \sqrt{\overline{\varphi'^2}} \tag{16}$$
- Also called RMS: $\varphi_{rms} \stackrel{\text{def}}{=} \sigma_\varphi$

Connection between time and ensemble average

Turbulence I.

Miklós Balogh

Preface

Definition

Notations

Statistics

Reynolds equations

Ergodicity

Time and ensemble averages are equal. I.e. the statistics are independent from the initial condition.

Average is the same, deviation... ?

$$\hat{\varphi}^{(T)} = \frac{1}{T}\int_0^T \varphi \quad dt \tag{17}$$

$$\overline{\hat{\varphi}^{(T)}} = \frac{1}{T}\int_0^T \overline{\varphi} \quad dt = \overline{\varphi} \tag{18}$$

Correlations

Covariance

$$\frac{R_{\varphi\psi}(x, y, z, t, \delta x, \delta y, \delta z, \tau) =}{\varphi'(x, y, z, t)\psi'(x + \delta x, y + \delta y, z + \delta z, t + \tau)}$$

Auto covariance

- If $\varphi = \psi$ covariance is called auto-covariance
- E.g. Time auto covariance:

$$R_{\varphi\varphi}(x, y, z, t, 0, 0, 0, \tau) \tag{19}$$

Correlation

Correlation

Non-dimensional covariance

$$\rho_{\varphi\psi}(x,y,z,t,\delta x,\delta y,\delta z,\tau) = \frac{R_{\varphi\psi}}{\sigma_{\varphi(x,y,z,t)}\sigma_{\psi(x+\delta x,y+\delta y,z+\delta z,t+\tau)}} \tag{20}$$

Integral time scale

Integral time scale

$$T_{\varphi\psi}(x,y,z,t) = \int_{-\infty}^{+\infty} \rho_{\varphi\psi}(x,y,z,t,0,0,0,\tau) \quad d\tau \qquad (21)$$

Taylor frozen vortex hypothesis

It is much more easy to measure the integral time scale (hot-wire) than the length scale (two hot-wires at variable distance)

Assumptions

- The flow field is completely frozen, characterised by the mean flow (U)
- The streamwise length scale can be approximated, by considering the temporal evolution of the frozen flowfield

Taylor approximated streamwise length scale

$$L^x = TU \tag{22}$$

Reynolds equations

We will develop the Reynolds average of the NS equations, we will call the Reynolds equations

Reynolds Averaged Continuity

The original equation

$$\partial_i u_i = 0$$

Development:

$$\begin{aligned} \overline{\partial_i u_i} &= \\ &= \partial_i \overline{u_i} \\ &= \partial_i \overline{\overline{u_i} + u_i'} \\ &= \partial_i \overline{\overline{u_i}} \\ 0 &= \partial_i \overline{u_i} \end{aligned} \tag{23}$$

Same equation but for the average!

Momentum equations

Derivation

- Same rules applied to the linear term (no difference only)
- Non-linear term is different

Averaging of the non-linear term

$$
\begin{aligned}
\overline{u_j \partial_j u_i} &= \\
&= \overline{\partial_j (u_j u_i)} \\
&= \partial_j \overline{u_j u_i} \\
&= \partial_j \overline{(\overline{u_j} + u_j')(\overline{u_i} + u_i')} \\
&= \partial_j \left(\overline{\overline{u_j}\,\overline{u_i} + \overline{u_i}\, u_j' + \overline{u_j}\, u_i' + u_j' u_i'} \right) \\
&= \partial_j \left(\overline{\overline{u_j}\,\overline{u_i} + u_j' u_i'} \right) \\
&= \partial_j \left(\overline{u_j}\,\overline{u_i} \right) + \partial_j \overline{u_j' u_i'} \\
&= \overline{u_j}\, \partial_j \overline{u_i} + \partial_j \overline{u_i' u_j'}
\end{aligned}
\tag{24}
$$

Reynolds equations

Continuity equation

$$\partial_i \overline{u_i} = 0$$

Momentum equation

$$\partial_t \overline{u_i} + \overline{u_j}\,\partial_j \overline{u_i} = -\frac{1}{\rho}\partial_i \overline{p} + \nu \partial_j \partial_j \overline{u_i} - \partial_j \overline{u_i' u_j'} \tag{25}$$

Reynold stress tensor

$$\overline{u_i' u_j'} \tag{26}$$

Or multiplied by ρ, or -1 times

Stresses

All stresses causing the acceleration

$$-\frac{1}{\rho}\overline{p}\,\delta_{ij} + \nu\partial_j\overline{u_i} \; - \overline{u_i' u_j'} \tag{27}$$

Questions?

Thanks for your attention!