Simple fluid dynamics problems

Lecture 2

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NavierâStokes equations

Conservation laws

- Momentum:

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\frac{1}{\rho}\nabla p + \nu \left[\nabla^2 \boldsymbol{u} - \frac{2}{3}\nabla\left(\nabla \cdot \boldsymbol{u}\right)\right] + \boldsymbol{g}$$

- Mass:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0$$

- Energy:

$$\frac{\partial\left(\rho c_p T\right)}{\partial t} + \nabla \cdot (\rho c_p T \boldsymbol{u}) = \nabla \cdot (k \nabla T) + H$$

Relationship between the material properties

- Ideal gas law:

$$p = \rho R T$$

Continous, general solution

A fundamental problem in analysis is to decide whether such smooth, physically reasonable solutions exist for the Navier–Stokes equations, thus the Clay mathematical institute posts 1 million dollar reward among the seven most important mathematical problems of the millennium. These are:

- Yang–Mills and Mass Gap
- Riemann Hypothesis
- P vs NP Problem
- Navier–Stokes Equation
- Hodge Conjecture
- Poincaré Conjecture (solved by Grigorij Perelman, 2003)
- Birch and Swinnerton-Dyer Conjecture

Numerical solution of the N–S equations

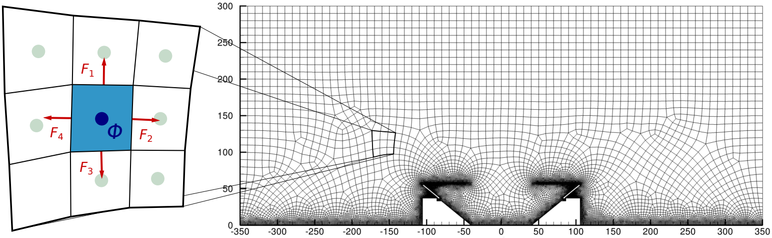
- While the analytical solution of the N–S equation are not known
  - Spatial discretization (mesh: grid or cell network)
  - Boundary conditions (at the bounding surfaces)
  - Temporal discretization (suitable time step, Δt)
  - Initial conditions (at $t = 0$)
- Simplification of geometry
- Simplifications of equations
  - Suitable coordinate system (Cartesian, cylindrical, spherical)
  - Steady vs. unsteady
  - Compressible vs. incompressible
  - Laminar vs. turbulent
  - External forces (gravitational, Coriolis, centripetal)

Numerical solution of the N–S equations

- Spatial discretization
  - Finite Volume Method (FVM)
  - Finite Element Method (FEM)
  - Finite Difference Method (FDM)
  - Spectral methods (e.g. for DNS on periodic domains)
  - Lattice gas model, lattice-Boltzmann method
- Temporal discretization (unsteady problems)
  - Explicit and implicit schemes, stability criteria (e.g. CFL)
  - Local time-step, adaptive time-step control
- Pressure-velocity coupling
  - Pressure correction (sequential, e.g. SIMPLE, PISO)
  - Coupled: simultaneous solution of the equations

Finite Volume Method (FVM)

- Arbitrary cells (volumes)
- Conservation laws are applied on these in integral form



Finite Volume Method (FVM)

- General form of the conservation laws:

$$\frac{\partial}{\partial t}\int_V \rho\phi \mathrm{d}V + \oint_A F\mathrm{d}\vec{A} = \int_V S_V \mathrm{d}V + \oint_A S_A \mathrm{d}\vec{A}$$

- Where ϕ and F respectively
  - The conservative quality per unit mass:

$$\phi = U/\rho$$

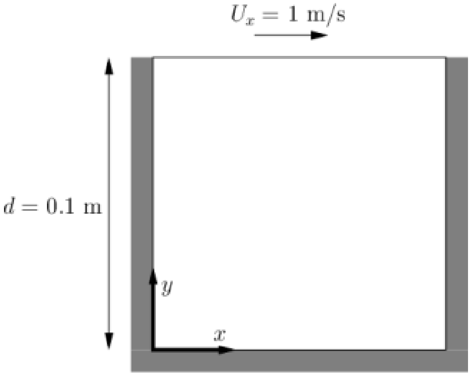
  - The sum of convective and conductive fluxes:

$$F = F_{konv.} + F_{kond.} = \rho\phi\vec{v} - \rho\nabla\phi$$

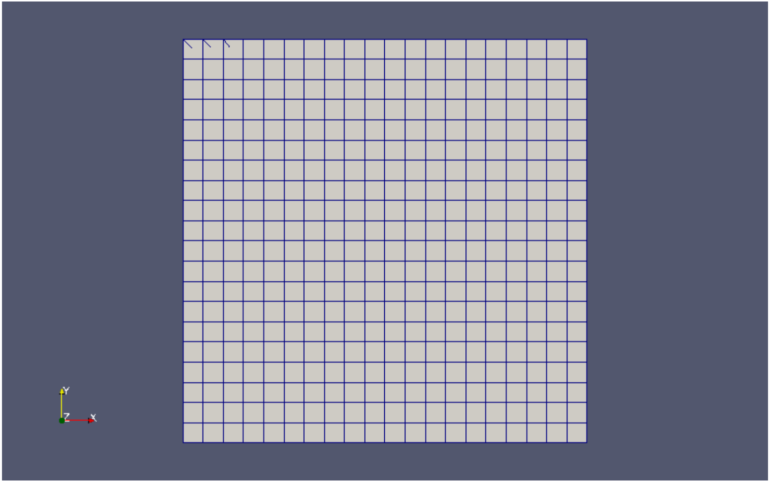
Steps of the numerical analysis

- Construction of the geometry (computational domain)
- Mesh generation
  - The basis of the spatial discretization
  - Decomposition of the domain to cells
- Definition of the boundary conditions
- Definition of the initial conditions
  - Constant - predefined values
  - Hybrid - potential flow solver
  - Patch - values given cell by cell (e.g. theoretical values)
- Simulation (numerical integration of the equations)
- Post-processing

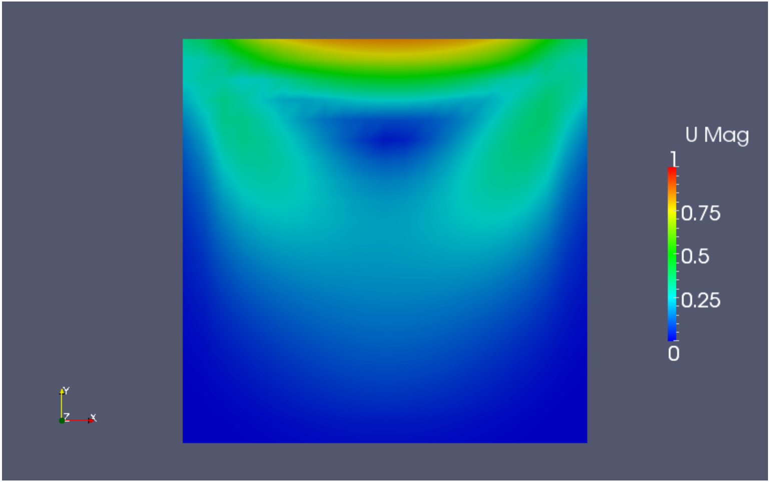
Lid-driven cavity – Geometry



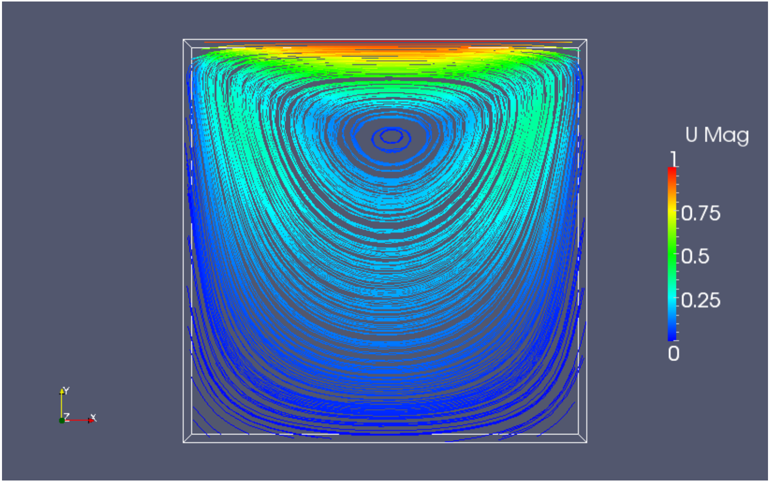
Lid-driven cavity – Mesh



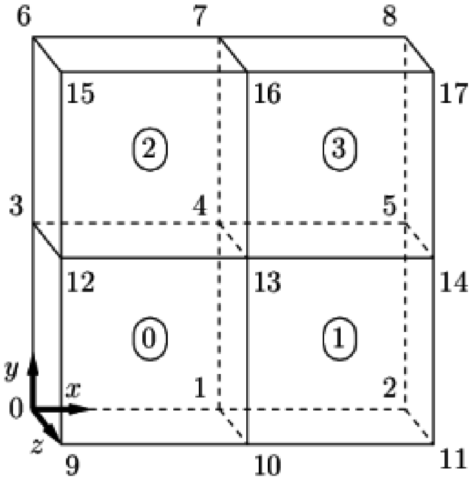
Lid-driven cavity – Velocity



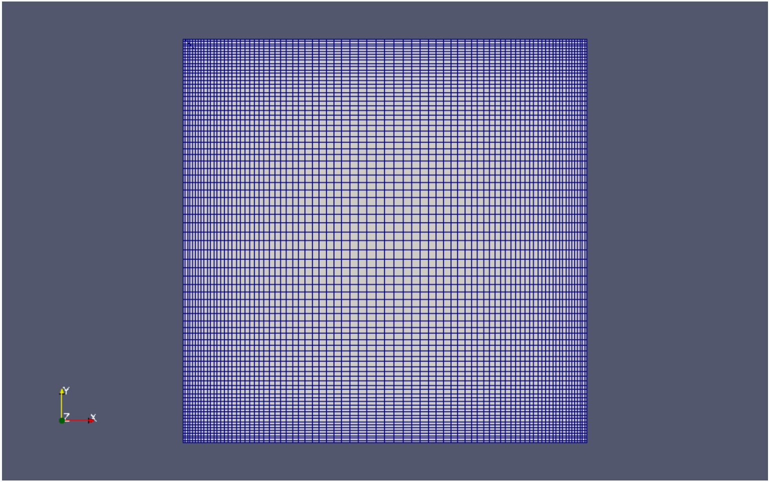
Lid-driven cavity – Streamlines



Refined lid-driven cavity – Geometry



Refined lid-driven cavity – Mesh



Refined lid-driven cavity – Velocity

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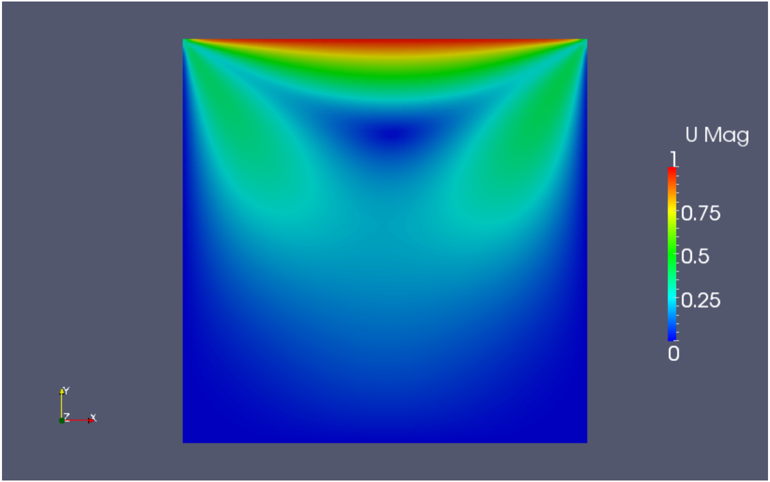

Refined lid-driven cavity – Streamlines

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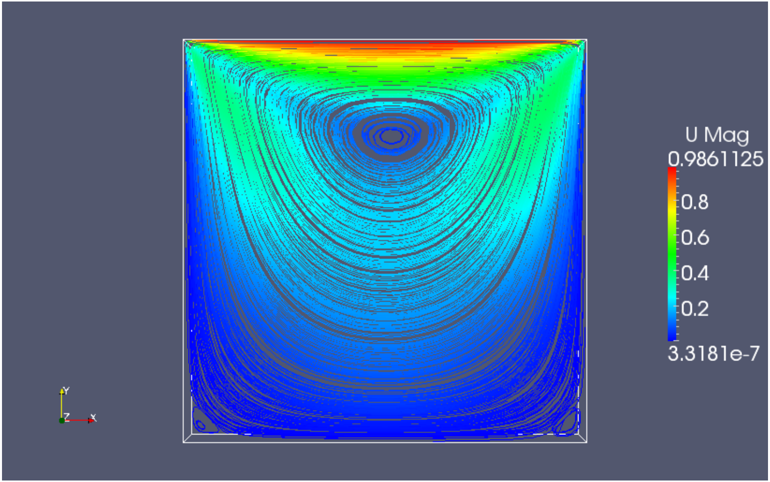
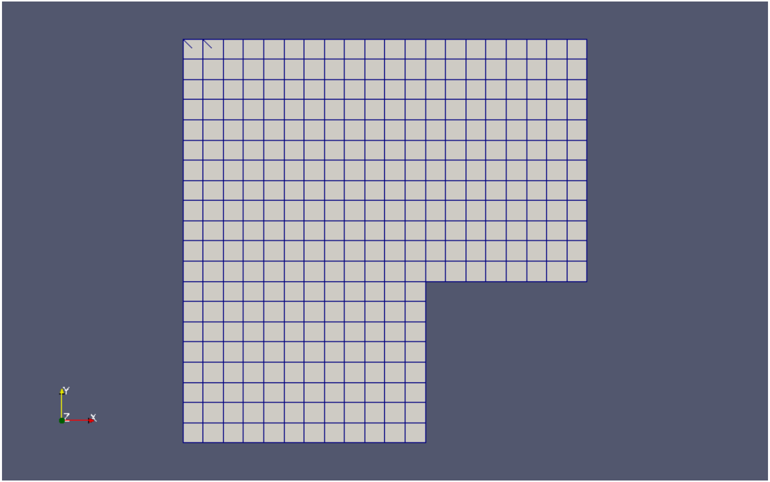
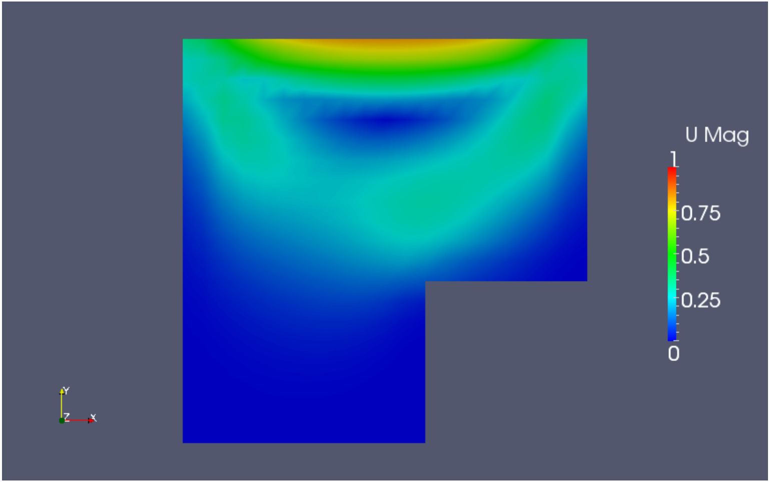


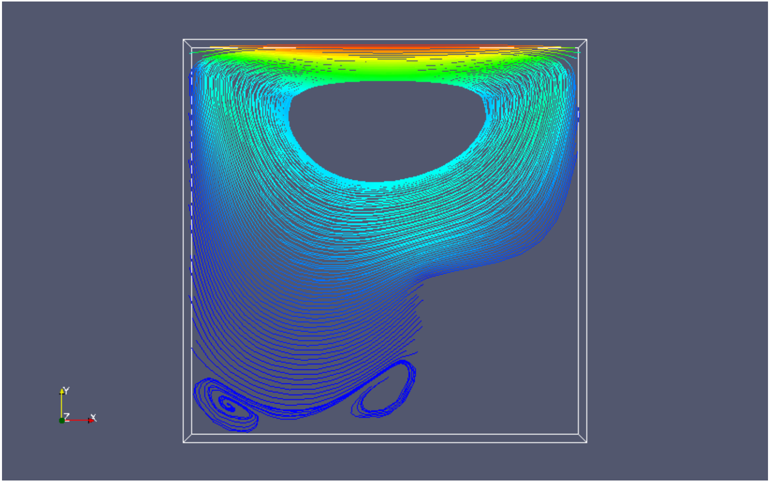
Clipped lid-driven cavity – Mesh



Clipped lid-driven cavity – Velocity



Clipped lid-driven cavity – Streamlines



Mapping fields in OpenFOAM

- One can initialize a simulation with former results
  - obtained even on lower resolution,
  - via interpolating the fields to the new mesh

```
cd $FOAM_RUN/tutorials/incompressible
cd icoFoam/cavity
blockMesh > blockMesh.log
icoFoam > icoFoam.log
cd ../cavityGrade
blockMesh > blockMesh.log
mapFields ../cavity -consistent
icoFoam > icoFoam.log
```

Bash scripts – executes linux commands in a row

Listing 1: Hello World sample script

```
#!/bin/bash
STR="Hello World!"
echo $STR
```

Listing 2: OpenFOAM runner sample script

```
#!/bin/bash
blockMesh > blockMesh.log
icoFoam > icoFoam.log
```

Bash scripts – executes linux commands in a row

Listing 3: Clocking sample script

```
#!/bin/bash
START_T=$(date +%s.%N)
# Do something time consuming here...
END_T=$(date +%s.%N)
ELAPS_T=$(echo "$END_T - $START_T" | bc)
```

Listing 4: Running a script

```
# Save as name.bsh and run with sh command
sh name.bsh
# Or just change permissions and run it
chmod +x name.bsh
./name.bsh
```

Questions?

Thanks for your attention!