



M02 INVESTIGATION OF A FREE JET

1. The free jet

A free jet refers to a flow which is entering a stationary field through an orifice. The volume flow rate of the jet is negligible as compared to the surrounding space. There are numerous engineering and industrial applications of free jets (e.g. ventilation equipment, refrigeration and jet powered machinery). For a detailed theoretical background of free jets, see Chapter 5 of Pope's book [1] (or Lesson 7.5 of the Áramlástan alapjai textbook in Hungarian [2]).



Figure 1: Examples of practical applications of free jets (left: Jet engine of the Eurofighter Typhoon F2 fighter plane, right: Water jet in a swimming pool)

2. Aim of the measurement

The aim of this measurement is to investigate the fluid dynamical characteristics of an air to air jet. In such jets, the fluid of both the stationary field and the jet is air, in particular, the air of the lab. The objective of this measurement is to measure the spacial structure of the jet, and to determine the velocity profile and the volume flow rate in different cross sections of the jet.

3. Measurement setup

The setup is shown in Fig. 2. The airflow is created by a centrifugal fan installed into the test rig, then outflows through a nozzle with circular cross section (4), creating the jet. The homogeneity of the velocity profile at the nozzle is provided by the flow straighteners and the screens installed before the outlet. The volume flow rate, and thus the velocity at the outlet – depending on the test rig – can be controlled by either the fan speed controller (5) or a throttle valve at the suction side. The shape of the jet is mainly determined by the geometry of the outlet nozzle and the volume flow rate. The velocity profile and the corresponding dynamic pressure profile can be measured with the Pitot-static tube (1) placed on an adjustable stand (3). The adjustable stand allows for vertical and horizontal positioning of the probe. The Pitot-static tube is connected to the digital manometer (2) using silicon tubes, from which the dynamic pressure can be read.

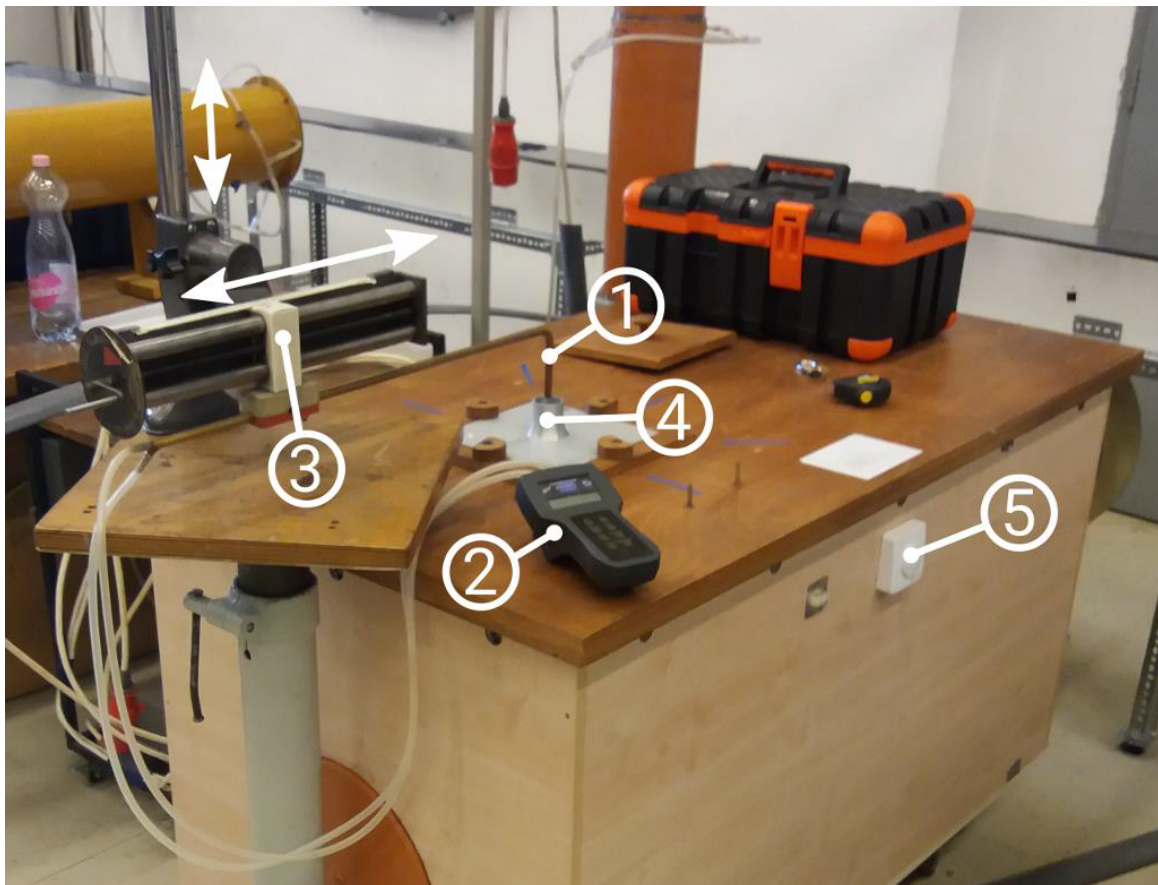


Figure 2: Measurement setup

4. Measurement procedure

4.1) Calibrate the digital manometer by comparing it to the Betz manometer!

If multiple digital manometers are used, perform the calibration on all of them!

4.2) Set the outlet velocity given in your individual measurement assignment!

Position the Pitot-static probe at the outlet, and measure the dynamic pressure corresponding to the maximal outlet velocity available on the test rig!

Calculate the dynamic pressure corresponding to the outlet velocity given in your measurement assignment, and tune the fan speed until you measure it at the outlet!

4.3) Map the flow field!

Measure the dynamic pressure p_{din} of the airflow at the heights z given in the measurement assignment with the corresponding Δr spacing! Record the location of the measurement points using the measurement height z and the position x of the Pitot-static probe read from the adjustable stand, the location of the jet axis is to be determined during post-processing!

Important: At a given height start and end the measurement in the stationary field ($p_{\text{din}} \leq 0$), using constant spacing and going through the middle of the jet!

4.4) Draw the verification diagram!

Plot the velocity profile of the free jet at the height given in the measurement assignment or by the lab instructor!

5. Post-processing and evaluation of the results

5.1) Perform the correction of the measured dynamic pressure values based on the calibration of the digital manometer!

Create a calibration diagram for the digital manometer, fit a linear trendline to the measured values! Give the equation of the trendline in the form

$$p_{\text{digital}} = a \cdot p_{\text{Betz}} + b, \quad (1)$$

and provide the correlation coefficient (R^2) of the fitting!

Using the equation of the trendline correct every pressure value measured with the digital manometer:

$$p_{\text{corr}} = M \cdot p_{\text{measured}} + p_{\text{offset}}, \quad (2)$$

where p_{corr} [Pa] is the corrected pressure value, p_{measured} [Pa] is the measured dynamic pressure, p_{offset} [Pa] is the offset error of the device, and $M = 1/a$ [-] is the slope of the line fit to the $p_{\text{Betz}}(p_{\text{digital}})$ data.

Important: If the device was zeroed before measurement, then $p_{\text{offset}} = 0$.

If multiple digital manometers were used, clearly denote which measurement points correspond to which manometer! Perform the correction step described above for each measured dynamic pressure value using the calibration of the corresponding manometer!

5.2) Determine the velocity profile at each height!

The velocity is given by

$$v = \sqrt{\frac{2}{\rho} p_{\text{dyn}}}, \quad (3)$$

where v [m/s] is the velocity at a given measurement point, p_{dyn} [Pa] is the corresponding corrected dynamic pressure, ρ [kg/m³] is the density of air:

$$\rho = \frac{p_0}{RT_0}, \quad (4)$$

here p_0 [Pa] is the atmospheric pressure, T_0 [K] is the ambient temperature, $R = 287$ [J/(kg · K)] is the specific gas constant of air.

5.3) Determine the position of the jet axis!

Since the jet axis does not necessarily fall on a measurement point, the obtained velocity profiles will not be perfectly symmetric. The aim of this task is to minimise the difference between the velocity profile and its mirrored version by translating the axis of reflection. The Appendix contains a short example of the procedure laid out here.

At a given height z let x_i be the i -th measurement point, v_i the corresponding velocity, where $i \in \{1, \dots, n\}$, and n is the number of measurement points. Let x_0 be the location of jet axis at height z .

Assuming that the velocity profile changes linearly between two measurement point (linear interpolation), the approximation of the profile between points x_i and x_{i+1} :

$$u_i(x) = \begin{cases} \frac{v_{i+1} - v_i}{\Delta r} x + \frac{v_i x_{i+1} - v_{i+1} x_i}{\Delta r}, & \text{if } x \in [x_i, x_{i+1}), \\ 0, & \text{otherwise.} \end{cases} \quad (5)$$

With this, the velocity profile approximated by a 1st order spline can be defined:

$$u(x) := \sum_{i=1}^{n-1} u_i(x) \quad (6)$$

It is easy to see that by reflecting the $u(x)$ profile across an axis \tilde{x} , we get the function $u(2\tilde{x} - x)$, this is shown in Fig. 3. Let the difference between the original and the mirrored profile be

$$Q(\tilde{x}) := \int_{-\infty}^{\infty} |u(x) - u(2\tilde{x} - x)|^2 dx. \quad (7)$$

(Note: The purpose of the square exponent is to give greater weight to points where the difference of the two profiles is larger. In practice – depending on the application – any $q \geq 1$ exponent may be used. For those interested: $Q(\tilde{x})$ is the L^2 distance of the two profiles, squared.)

The jet axis is the x_0 axis, across which reflecting the $u(x)$ profile yields the minimal $Q(x_0)$, meaning

$$Q(x_0) = \min \{Q(\tilde{x}) : \tilde{x} \in \mathbb{R}\}. \quad (8)$$

Determine the position of the jet axis x_0 with 0.5 [mm] precision at every z measurement height! Moving forward, use the (r_i, v_i) data points instead of the (x_i, v_i) data points, where $r_i = x_i - x_0$!

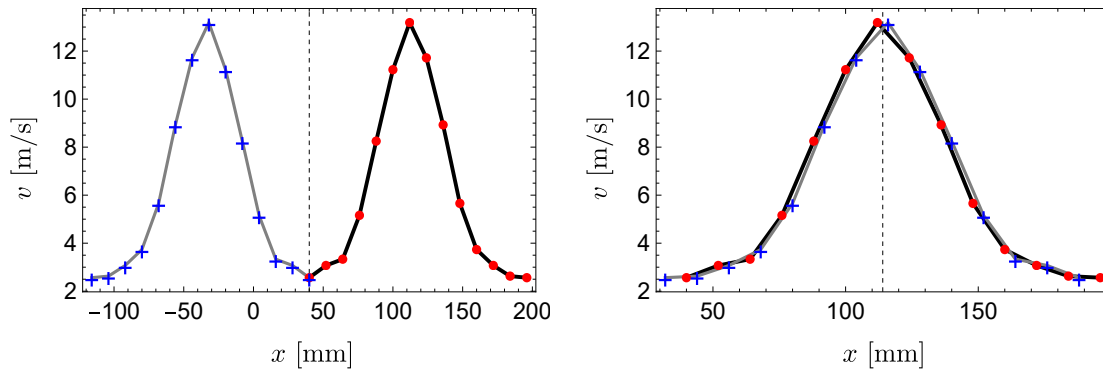


Figure 3: The $u(x)$ (black) and the reflected $u(2\tilde{x}-x)$ (grey) velocity profiles. The measurement data and the data reflected across axis \tilde{x} is denoted by red dots and blue crosses, respectively. On the left diagram $\tilde{x} = 40$ [mm], on the right $\tilde{x} = 114$ [mm] $\approx x_0$, whose positions are shown with vertical dashed lines.

5.4) Plot the measured $v(r)$ velocity profiles at each height z in a common diagram with error bars!

The details of the uncertainty calculation are given in Section 6.

5.5) Plot the maximum velocity of the measured velocity profiles as a function of height: $v_{\max}(z)$

5.6) Determine the core distance of the free jet!

By definition, the z_{core} core distance:

$$v_{\max}(z_{\text{core}}) = 0.95v_{\max}(0), \quad (9)$$

meaning the distance where the maximum velocity of the profile decreases to 95% of the outlet velocity. Use linear interpolation of the $v_{\max}(z)$ points for the calculation!

Express the core distance with the outlet diameter D_0 ! Compare the constant factor obtained with values from literature!

5.7) Plot the dimensionless velocity profiles $v'(r')$ at each height z in a common diagram!

At height z , the v flow velocity should be normalised by the maximum velocity $v_{\max}(z)$ of the corresponding velocity profile, whereas the distance r from the jet axis should be normalised by the distance $r_{1/2}$ corresponding to half of the maximum velocity of the profile!

Meaning:

$$v' := v/v_{\max}, \quad r' := r/r_{1/2} \quad (10)$$

Use linear interpolation between measurement point to accurately determine the value of $r_{1/2}$!

Important: Determine the distance corresponding to half of the maximum velocity on both sides of the jet axis ($r_{1/2+}$, $r_{1/2-}$), and use their average for normalisation:

$$r_{1/2} = \frac{|r_{1/2+}| + |r_{1/2-}|}{2} \quad (11)$$

Compare the dimensionless velocity profiles after the core distance!

5.8) Determine and plot the volume flow rate q_v as a function of height!

At height z , the volume flow rate q_v could be exactly calculated by integration given a perfect knowledge of the $v(x, y)$ velocity profile:

$$q_v = \iint_{\mathbb{R}^2} v(x, y) dx dy \quad (12)$$

Lacking this, numerical integration has to be used, for which we will make two approximations. As the measurements were done in a single vertical plane, we assume that the v velocity profile is constant along semi-circles whose midpoint lies on the measurement plane, thus it is a function of the coordinate $r = x - x_0$ only:

$$v(x, y) = \tilde{v}(r, \varphi) = \tilde{v}(r) \quad (13)$$

(Note: Notice that this is not the usual polar coordinate system, as r may be negative due to the translation by x_0 . This is consistent with the observation that $Q(x_0) \neq 0$, meaning the measured values are not perfectly symmetrical to the axis x_0 .)

Our second assumption is that

$$\tilde{v}(r) = v_i, \quad \text{if } r \in \left[r_i - \frac{\Delta r}{2}, r_i + \frac{\Delta r}{2} \right), \quad (14)$$

which means the velocity in the measurement plane is constant in a Δr wide neighbourhood of the measurement point (rectangle method). With these, the numerical integration for the volume flow rate:

$$q_v = \sum_{i=1}^n v_i A_i, \quad (15)$$

where A_i is the area of the region in which $v(x, y) = v_i$ is constant according to our assumptions. The A_i areas obtained from the assumptions is shown in Fig. 4., where j is the index corresponding to the closest measurement point to the axis x_0 , meaning

$$0 \in \left[r_j - \frac{\Delta r}{2}, r_j + \frac{\Delta r}{2} \right). \quad (16)$$

Based on these, it is easy to see that

$$\begin{aligned} A_i &= \pi |r_i| \Delta r, \quad i \neq j, \\ A_j &= \frac{\pi}{2} \left(r_j + \frac{\Delta r}{2} \right)^2 + \frac{\pi}{2} \left(r_j - \frac{\Delta r}{2} \right)^2 = \pi r_j^2 + \frac{\pi}{4} \Delta r^2. \end{aligned} \quad (17)$$

Perform the numerical integration at each height z , and plot the obtained $q_v(z)$ function! Fit a linear trendline to the data points, and determine its equation along with the corresponding correlation coefficient!

5.9) Plot the $q'_v(z')$ dimensionless volume flow rate!

Normalise the volume flow rates by the volume flow rate $q_v(0)$ measured at the outlet, and the height by the outlet diameter D_0 !

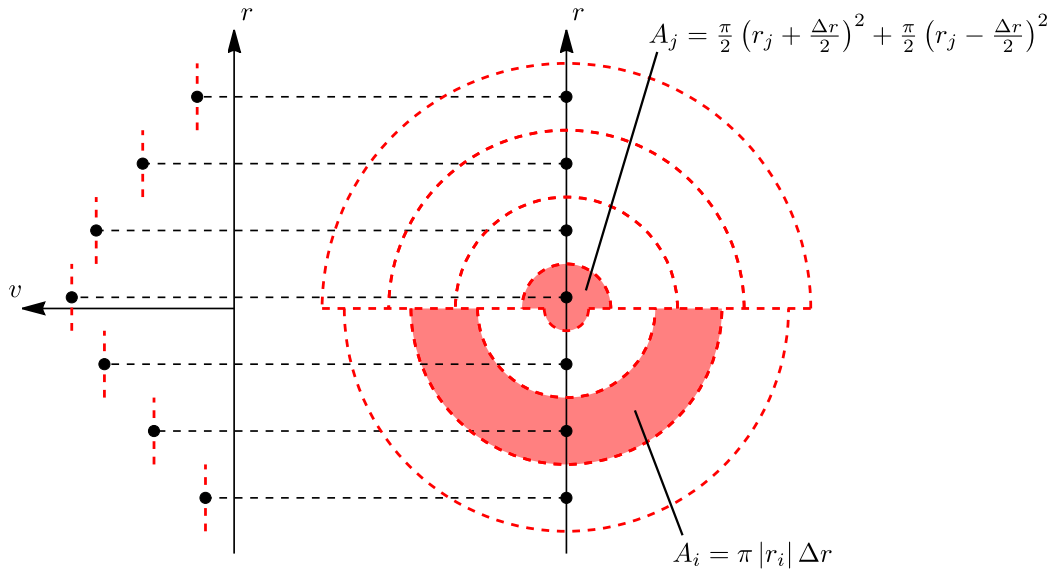


Figure 4: The measured $\tilde{v}(r)$ function (left) and the areas corresponding to the measurement points (right), assuming uniform Δr spacing. The measurement points are shown by black dots.

5.10) Determine and plot the thrust F generated by the free jet as a function height!

At height z the thrust can be calculated similarly to the volume flow rate $q_v(z)$ using a surface integral:

$$F = \rho \iint_{\mathbb{R}^2} v^2(x, y) dx dy \quad (18)$$

Using the same assumptions as in section 5.8), the integral in Eq. (18) can be approximated as

$$F = \rho \sum_{i=1}^n v_i^2 A_i, \quad (19)$$

where A_i is the surface where the velocity $v(x, y) = v_i$ is assumed to be constant. The exact expression for these A_i surfaces is given in Eq. (17).

Perform the numerical integration at every measurement height z and plot the resulting $F(z)$ function! Fit a linear trendline to the data, and determine its equation along with the corresponding correlation coefficient!

6. Measurement uncertainty calculation

The uncertainty calculation has to be performed based on the Gaussian error propagation for each measurement point of the velocity profiles. According to this, the δR measurement uncertainty of a quantity R dependent of quantities X_1, \dots, X_k can be calculated as

$$\delta R = \sqrt{\sum_{j=1}^k \left(\frac{\partial R}{\partial X_j} \cdot \delta X_j \right)^2}. \quad (20)$$

In this task, the measurement uncertainty of the velocity at the measurement points has to be calculated, thus $R = v_i$. The directly measured quantities (X_j) which are subject to measurement error, and their measurement uncertainties (δX_j):

- | | | |
|---|--------------------------|---|
| 1) Dynamic pressure measured with Pitot-static probe: | $X_1 = p_{\text{dyn},i}$ | $\delta X_1 = \delta p_{\text{dyn},i} = 2$ [Pa] |
| 2) Atmospheric pressure: | $X_2 = p_0$ | $\delta X_2 = \delta p_0 = 100$ [Pa] |
| 3) Ambient temperature: | $X_3 = T_0$ | $\delta X_3 = \delta T_0 = 1$ [K] |

6.1) Express the pointwise velocity v_i such that the expression contains only quantities directly measured by you, and constants with known values!

6.2) Determine the $\partial v_i / \partial X_j$ partial derivatives!

6.3) Calculate the absolute (δv_i) and relative ($\delta v_i / v_i$) uncertainty of the velocity at each measurement point using Gaussian error propagation!

Apply error bars to the velocity profiles plotted in a common diagram according to the obtained absolute error values (see Task 5.4)!

Remarks

Be aware of the following during the measurement:

- Before turning any measurement device on or in general during the lab, make sure that safe working conditions are ensured. The other participants have to be warned of the starting of the machines and of any changes that could endanger the members of the lab.
- The atmospheric pressure and room temperature should be recorded before and after every measurement.
- The measurement units and other important factors (e.g. data sampling frequency, date of calibration) of every recorded value of the applied measurement devices should be recorded.
- Type and construction number of the applied measuring instrument should be included in the final report.
- Checking and harmonizing of the units of the recorded values with those used in further calculations.
- The digital manometer must be calibrated.
- The measurement ports of the pressure meter should be carefully connected to the correct pressure ports of the instrument.
- If inlet or outlet tubes are to be assembled with fans, connections should be airtight as escaping/entering air can significantly modify the measurement results.
- The measurement ports of the pressure meter should be carefully connected to the correct pressure ports of the measurement instrumentation. The integrity of the silicone tubes used for connection must be verified before and during the measurement (no holes, slits, etc.).

Before submission of your report, verify that the document satisfies the requirements regarding content and formatting laid out in the [List of the lab report requirements](#)! It is recommended to ask for a consultation before submitting the report if you have any questions or problems.

Ensure that even the suspicion of plagiarism concerning the lab report is avoided! When taking any visual or written content (with or without modification) from other sources – including the present measurement guide – the original source must be indicated by a proper reference!

Appendix: Determining the jet axis

Let the measured velocity profile be the one given in Table A.1. Then we can approximate the velocity profile between data points as

$$u_i(x) = \begin{cases} a_i x + b_i, & \text{if } x \in [x_i, x_{i+1}), \\ 0, & \text{otherwise,} \end{cases} \quad (\text{A.1})$$

where

$$\begin{aligned} a_i &= \frac{v_{i+1} - v_i}{\Delta r}, \\ b_i &= \frac{v_i x_{i+1} - v_{i+1} x_i}{\Delta r}. \end{aligned} \quad (\text{A.2})$$

The obtained a_i and b_i values are also given in Table A.1.

x_i [m]	0.03	0.04	0.05	0.06	0.07
v_i [m/s]	0	2.58	3.77	2.91	0
a_i [1/s]	258	119	-86	-291	-
b_i [m/s]	-7.74	-2.18	8.07	20.37	-

Table A.1: Example velocity profile data

According to task **5.3**) the x_0 jet axis has to be determined with a 0.5 [mm] accuracy. For the sake of this example, let us divide the region between x_1 and x_n with a step size of $\Delta x = 0.1$ [mm]. The obtained points will be used to approximate $Q(\tilde{x})$.

In Excel, the $u_i(x)$ interpolation can easily be calculated for a given x using the built-in IF() command, thus it can be quickly determined for $N = 400$ points using the autocomplete feature. Table A.2 shows an example of this for $\tilde{x} = 0.05$ [m] along with the $u(x)$, $u(2\tilde{x} - x)$ profiles, and their squared difference.

x [m]	0.0300	0.0301	...	0.0699
$u_1(x)$ [m/s]	0	0.0258	...	0
\vdots	\vdots	\vdots	\ddots	\vdots
$u_4(x)$ [m/s]	0	0	...	0.0291
$u(x)$ [m/s]	0	0.0258	...	0.00291
$2\tilde{x} - x$ [m]	0.0700	0.0699	...	0.0301
$u_1(2\tilde{x} - x)$ [m/s]	0	0	...	0.0258
\vdots	\vdots	\vdots	\ddots	\vdots
$u_4(2\tilde{x} - x)$ [m/s]	0	0.0291	...	0
$u(2\tilde{x} - x)$ [m/s]	0	0.0291	...	0.00258
$ u(x) - u(2\tilde{x} - x) ^2$	0	$1.089 \cdot 10^{-5}$...	$1.089 \cdot 10^{-5}$

Table A.2: The calculation steps for $Q(\tilde{x})$ with $\tilde{x} = 0.05$ [m]

From this, $Q(\tilde{x})$ can be determined by summing the last row of the table (and multiplying it by Δx). The result of this procedure is shown in Table A.3. for different \tilde{x} values. Clearly, the minimum of $Q(\tilde{x})$ occurs at

$$x_0 = 0.0505 \text{ [m]}, \quad (\text{A.3})$$

meaning this is the location of the jet axis.

\tilde{x} [m]	$Q(\tilde{x})$
0.0495	0.558
0.0500	0.145
0.0505	0.061
0.0510	0.282

Table A.3: $Q(\tilde{x})$ for a few different values of \tilde{x}

References

- [1] Stephen B. Pope, Turbulent Flows, Cambridge University Press, 2000
- [2] Lajos Tamás, Áramlástan alapjai, Műegyetemi Kiadó, 2015