STUDY ON THE STRAIN FIELD IN THE WAKE OF AN AIRFOIL

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ABSTRACT

This paper shows an initial study on the role of the strain, i.e. the symmetric part of the velocity gradient tensor, in the formation of the cascade of the turbulent scales. The present investigation is based on post-processing existing data produced by means of Large Eddy Simulation. When turbulent flow is to be analyzed, vortices and the dynamics of vortices are usually focused on. The turbulent energy cascade is usually explained with the aid of the induced velocity field of the vortices expressed by the Biot-Savart law. However the initial disturbances that leads to the deformation of the vortex filaments, as well, as the stretching mechanism of them is related to the strain field. The flow field around a wing section was determined and the eigenvalues and eigenvectors of the strain tensor were calculated for a time instant. Iso-surfaces of the eigenvalues of the strain tensor are compared to those of the Q criterion.

1. INTRODUCTION

The behavior of the vortices, that rolled up from a shear layer are characterized by severe three-dimensional instability leading to their deformation and break-up. The mechanism of the brake-up of the initially well organized vortex tubes is based on the induced velocity field of the vortex filaments [1,2].

A plane shear layer is known to be unstable for any upstream disturbances. The appearance of the so called Kelvin-Helmholtz waves is a primary instability, which basically means the reorganization of the originally evenly distributed vorticity into regions of high vorticity (cores) and regions of low vorticity or high strain (braids). The flow, characterized by spanwise vortices is two-dimensional in this stage, but it may go under further development.

A possible way of the flow development is suggested by Lasheras and Choi [2]. According to them a severe three-dimensional instability originates in the braids. The small upstream disturbances deform the weak vortex tubes in the braids, which under the effect of the strong strain field, start to be stretched in the direction of the flow. Due to mutual interaction of the weak and the strong vortex tubes, the strong ones get wavy and the weak ones turn in the direction of the flow, by this creating a system of streamwise vortices superimposed on the spanwise ones. Whenever the streamwise vortices appear in the core zone, they go through further interaction, which finally results in the break-up of the coherent structures.

As the strong strain field in the braids is said to play a key role in the appearance of the streamwise structures, characteristics of the strain tensor will be examined later on in this paper.

2. DESCRIPTION OF THE COMPUTATIONAL MODEL

The finite volume method based CFD code computation has been carried out in three domains of different lengths given by the non-dimensional parameter of chord length (c) about flow pattern characterized with Reynolds number of 1.22.105 around a RAF6 type airfoil [3]. The airfoil positioned 0.5c from the inlet. The dimensions of the domains are 3cx1cx1c, 3cx1cx0.125c and 3cx1cx0.125c length stream wise, wall-normal direction and spanwise direction, respectively. The domains bounded with path lines computed from previous RANS simulation with k-e turbulent model in airfoil wall normal direction [4]. Each domain was meshed using O-H type structured grid using 2.006.150 hexahedral cells. The mesh was appropriately refined in the direction to the walls of the airfoil by an expansion ratio of approximately 7% to enable the accurate resolution of the boundary layer. The wall normal size of the first cells attaching to the airfoil is linearly increasing along the chord (both on the suction and pressure side) starting from 10^{-4}c to 2.10^{-4}c height. This resolution corresponds to cell sizes in wall unit (y^{+}) less than 1 in 99.6% of the domain. The equi-angle skew of none of the cells exceed the value of 0.67, which is appropriate for the numerical schemes used in this study. The FLUENT 6.2 solver [5] was applied using cell centered collocated variable arrangement, implemented for unstructured grid. For this constant density simulation segregated solver was used for the sequential solution of the governing equations. Second-order upwind discretization scheme was applied for the pressure while for the momentum the bounded central differencing scheme (BCD) was chosen. The pressure-velocity coupling was satisfied with fractional step method (FSM) [6,7] in agreement with the applied non-iterative time-advancement (NITA) for the transition control of the solver [5]. To solve the equations and calculate the turbulence phenomena the Smagorinsky-Lilly based large-eddy simulation was used with dynamic stress model [8,9] with a time-step of $3.5.10^{-6}$s.

The final judgment of the errors relied on monitoring of representative physical parameter like the drag coefficient and lift coefficient and the area-weighted averaged kinetic energy in the domain as well.

The applied boundary conditions can be seen in **Figure 1**. At the inlet boundary uniform streamwise velocity (calculated from the 2D RANS calculation [4]) without any perturbation algorithm modeling fluctuating velocity algorithm was applied. The pressure at the boundaries can be interpolated from the domain. The wall of the airfoil was modeled as no-slip wall. For the outflow section, homogenous Neumann conditions were applied for velocity and pressure. Specified pressure gradient periodic condition was applied in span-wise direction to model the three dimensional flow around the airfoil.

The simulations were compared with in-house hot-wire, laser Doppler anemometry and pressure measurements in an NPL type wind tunnel. The bases of the comparison were six velocity profiles perpendicular to the suction side and three profiles perpendicular to the pressure side and three profiles in the wake, 0.1c, 0.25c and 0.5c behind the airfoil.

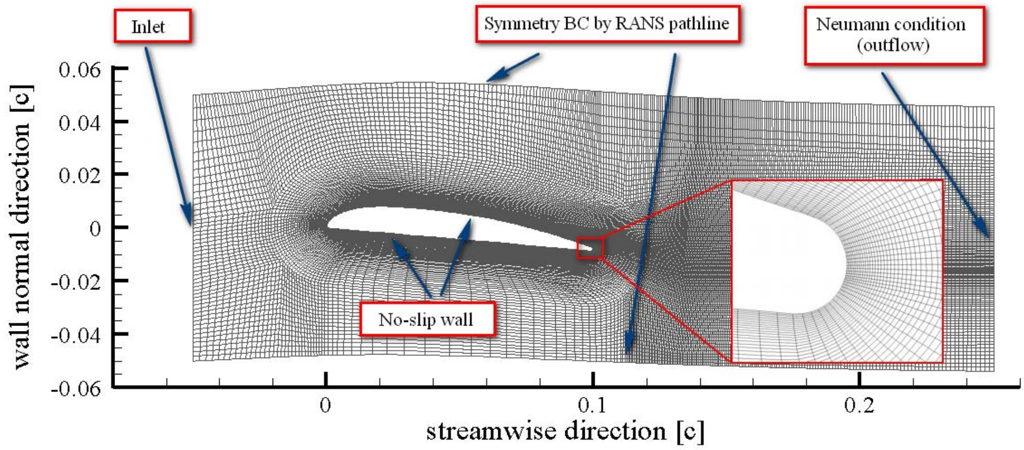


Figure 1. Applied boundary conditions in the LES

The pressure was measured on both surfaces of the airfoil in 17 pressure taps. In both parameters the LES results have shown better agreement than RANS with k-ε and k-ω turbulence model [4]. But the pressure-coefficient is still over predicted by simulation.

3. THE STRAIN TENSOR AND ITS EIGENVALUES

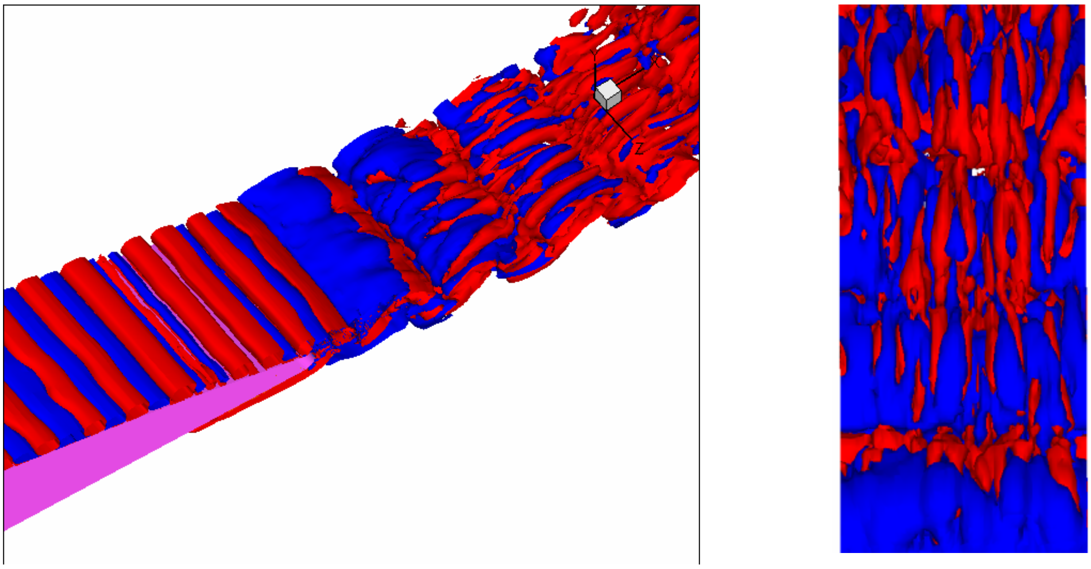
From basic considerations of fluid mechanics it has been derived that strain plays key role in the mechanics and energy budget of fluid flow, as it has strong relation with the viscous forces and energy dissipation. The definition of the strain tensor can be seen in Eq.(1).

$$S_{ij} = \frac{1}{2}\left(\partial_j u_i + \partial_i u_j\right), \quad (1)$$

where S_{ij} is an element of the strain tensor, u_i [m/s] is the i[th] component of the instantaneous velocity ($u_1 = u_x, u_2 = u_y; u_3 = u_z$ using the notation of Cartesian coordinates), i, j =1,2,3. The trace of this tensor is the equation of continuity for incompressible flows. It can be shown that the second invariant is equivalent to the dissipation rate of the turbulent kinetic energy [10]. As the tensor of strain is symmetric and all elements of it are real, it has three real eigenvalues and three orthogonal eigenvectors. It is trivial that in two-dimensional flow, the strain tensor has two eigenvectors, out of which, the first is oriented in 45° to the tangent of the streamlines, while the second is oriented orthogonally in 135° to them. In three-dimensional flows, there are three eigenvectors and the corresponding three eigenvalues are usually denoted and called as follows: α elongational, β intermediate, γ compressional. Due to the continuity for incompressible flows, $\alpha > 0$, $\gamma < 0$ and β can be either negative, or positive. The intermediate eigenvalue, β, is zero for two-dimensional flow or at least, for two-dimensional strain state. If $\beta > 0$ then the fluid element is stretched in the third direction and the element takes a *sheet-like* shape, while in case of $\beta < 0$ the element is compressed and it takes *tube-like* shape [10, 11]. The eigenvalues and eigenvectors of the strain-rate tensor were computed based on the Jacobi transformation for symmetric matrices [12]. In the following section a time instant of the LES computation is analyzed.

4. DESCRIPTION OF THE FLOW FIELD, VORTEX AND STRAIN STRUCTURES

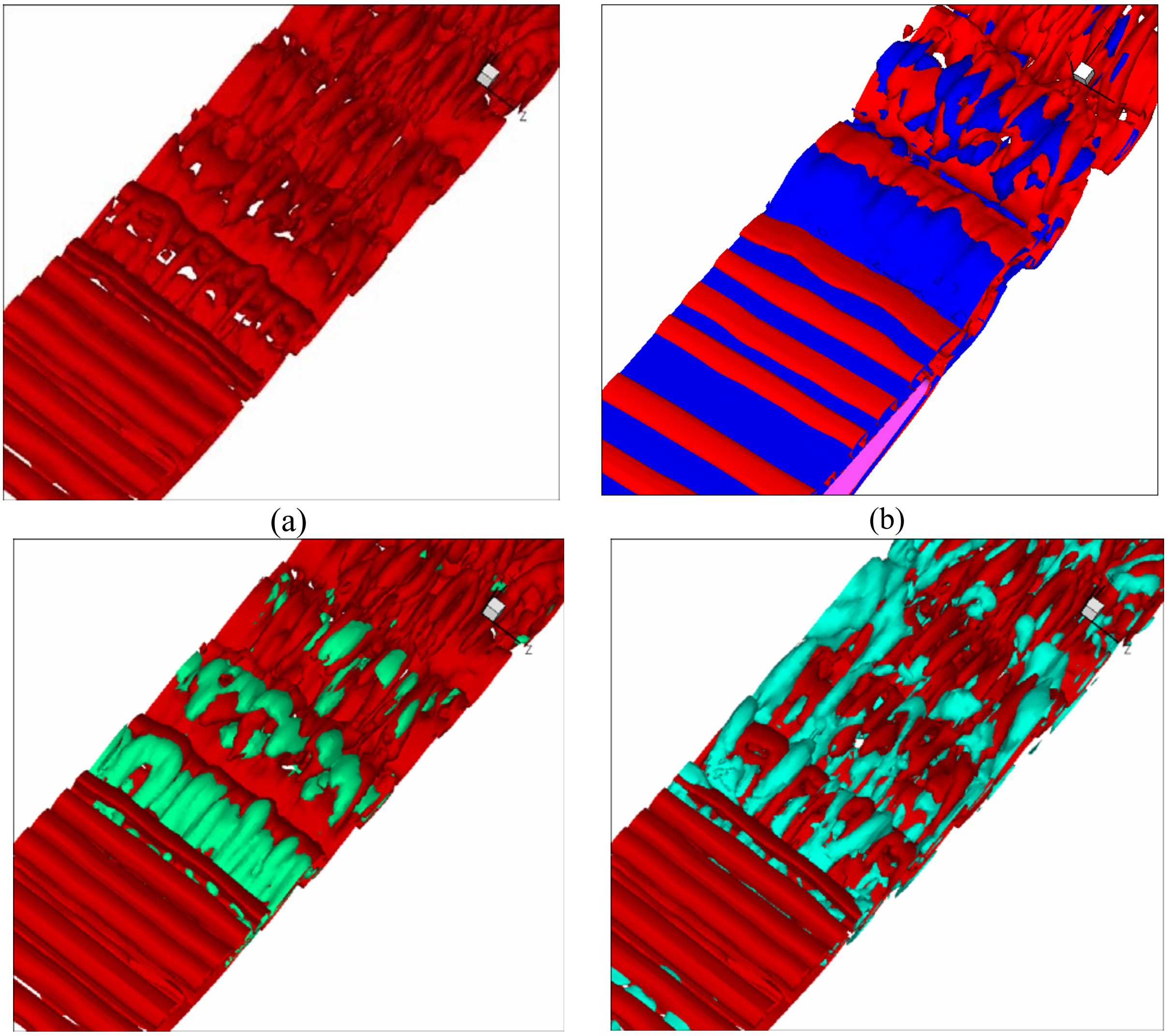
An effective way to examine the structure of a turbulent flow is to analyze the existing vortices and their dynamics. For the visualization of vortices in the investigated flow, the Q-criterion was used (**Figure 2**). The light color stands for positive Q-isosurfaces, when vorticity dominates over strain, while the dark color stands for negative Q-isosurfaces, when strain dominates over vorticity. From **Figure 2a** it can be observed that the flow on the suction side of the wing is purely characterized by spanwise vortices. In the first three main structures of the wake spanwise vortices still dominate, but later streamwise vortices appear, superimposed on the spanwise ones (**Figure 2b**). Another important observation is that strong strain field appears exactly between two vortices, which can be clearly seen on the wing and somewhat less clearly in the wake. Both the structure of the flow field and the location of the strong strain field is in good accordance with the formerly mentioned observations of Lasheras and Choi [2].



(a) (b)

Figure. 2 Positive and negative Q-isosurfaces (a) on the wing, (b) in the wake

To find the role of the strain field in the evolution of the flow structure, the Q-isosurfaces and the isosurfaces of the three eigenvalues of the strain-rate tensor were compared. **Figure 3a** shows only the Q-isosurfaces, as a basis of comparison, while on **Figure 3 b**, **c** and **d** it appears together with first, the positive second and the negative second eigenvalues, respectively. Based on previous statements the first (elongational) eigenvalue is expected to be situated between high vorticity regions. This is supported by the second big structure of **Figure 2b**. The first structure seems to contradict this statement, however it behaves on the same way, only the iso-value corresponding the visible organization of the structures is higher than in case of the represented one. The first and the third eigenvalues were found to be approximately coincident, and of the same order.



(a) (b) (c) (d)

Figure. 3 (a) Q-isosurfaces (b) Q-isosurfaces (light) with first eigenvalue (dark color) (c) Q-isosurfaces (dark color) with second (light color), positive eigenvalue (d) Q-isosurfaces with second (light color), negative eigenvalue

The second eigenvalues also seem to have a strong relationship with the Q-structures of the flow. **Figure 3c** shows that right after the trailing edge the positive second eigenvalue coincides quite well with the Q-isosurfaces, from which we can suppose, that regions of high vorticity tend to be sheet-like. From **Figure 3d** it turns out that negative second eigenvalues mainly appear between the Q-structures, which suggests a tube-like deformation of fluid elements there. This arrangement of sheet-like and tube-like elements seems to be logical, as if two elements are stretched in the x-z plane, the element between them has to be compressed in the z-direction and elongated in the x-direction because of its constant volume. The first and third eigenvalues are one order of magnitude higher than the second ones that indicates two dimensional strain in the three dimensional flow field.

CONCLUSIONS

The structure of the iso-surfaces of the eigenvalues of the strain tensor was shown in this paper using the flow in the wake of an airfoil, computed by means of Large-Eddy simulation. The most significant components of the strain can be related to its first and third eigenvalues, the iso-surfaces of which, show some overlap between regions of high strain and high vorticity. This overlap might play a role in the vortex stretching and vortex deformation that finally governs the turbulent cascade process. It was found that the strain was mainly two dimensional in the three dimensional flow field which is concluded from the low value of the second eigenvalue that is responsible for three dimensional strain effects. The iso-surfaces belonging to the positive and negative values of the second eigenvalue of the strain tensor showed counter-phase arrangement. The sheet-like deformation occurs mainly in the high vorticity regions, while tube-like deformation is placed between the vortices. The sheet-like deformation might indicate a way of more effective specific area increase that can be important for multi-component reactive flows.

ACKNOWLEDGEMENT

Authors of this paper wish to express their thanks for the LES group of the Deparment of Fluid Mechanics for providing the flow field to be analyzed here. Special thanks to Péter Tóth for the implementation of the algorithm for the computation of the eigenvalues and eigenvectors [12] into FLUENT.

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